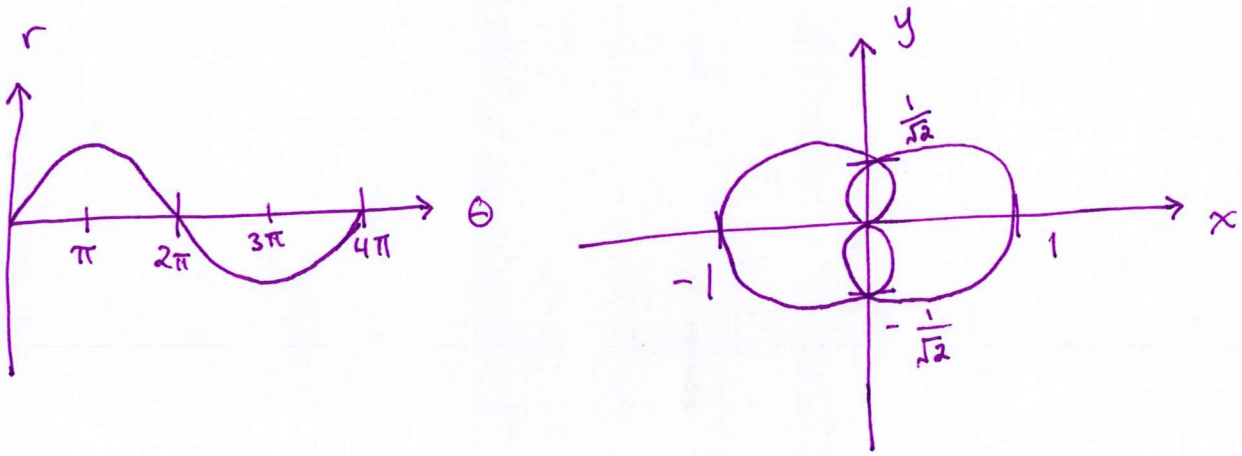
Math 227 Sections 29 and 33 Quiz 6 3/23/18

Name:_____

1. Sketch the curve $r = \sin(\theta/2)$ by first sketching the graph of r as a function of θ in Cartesian coordinates. You'll need to graph it from $\theta = 0$ to $\theta = 4\pi$. Label your axes.



2. Set up an integral that gives the arc length of the curve $r = \sin(\theta/2)$.

$$\frac{dr}{d\theta} = \frac{1}{2}\cos\left(\frac{\theta}{2}\right)$$

$$L = \int_0^{4\pi} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta = \boxed{\int_0^{4\pi} \sqrt{\left(\sin\left(\frac{\theta}{2}\right)\right)^2 + \left(\frac{1}{2}\cos\left(\frac{\theta}{2}\right)\right)^2}\, d\theta}$$

3. Find the slope of the tangent line to the curve $r = \sin(\theta/2)$ at $\theta = \pi/3$.

$$\frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

$$r\Big|_{\frac{\pi}{3}} = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

$$\frac{dr}{d\theta}\Big|_{\frac{\pi}{3}} = \frac{1}{2}\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{4}$$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{3}} = \frac{\frac{\sqrt{3}}{4}\cdot\sin\left(\frac{\pi}{3}\right) + \frac{1}{2}\cdot\cos\left(\frac{\pi}{3}\right)}{\frac{\sqrt{3}}{4}\cdot\cos\left(\frac{\pi}{3}\right) - \frac{1}{2}\sin\left(\frac{\pi}{3}\right)} = \boxed{\frac{\frac{\sqrt{3}}{4}\cdot\frac{\sqrt{3}}{2} + \frac{1}{2}\cdot\frac{1}{2}}{\frac{\sqrt{3}}{4}\cdot\frac{1}{2} - \frac{1}{2}\cdot\frac{\sqrt{3}}{2}}}$$