

Math 226 Sections 29 and 33 Quiz 4 (make up)

Name: _____

If an integral is proper, evaluate it. If an integral is improper, either show that it diverges or evaluate it.

$$1. \int \frac{3t-2}{t+1} dx = \int \frac{3t+3-5}{t+1} dt = \int \frac{3(t+1)}{t+1} - \frac{5}{t+1} dt$$

$$= \int 3 - \frac{5}{t+1} dt = \boxed{3t - 5 \ln |t+1| + C}$$

$$2. \int_0^2 \frac{1}{(4+x^2)^{3/2}} dx$$

See Quiz 4 solutions.

$$3. \int_0^5 \frac{1}{\sqrt[3]{5-x}} dx = \lim_{t \rightarrow 5^-} \int_0^t \frac{1}{\sqrt[3]{5-x}} dx = \lim_{t \rightarrow 5^-} \left. -\frac{3}{2} (5-x)^{\frac{2}{3}} \right|_0^t$$

$$= \lim_{t \rightarrow 5^-} \left(-\frac{3}{2} (5-t)^{\frac{2}{3}} + \frac{3}{2} (5)^{\frac{2}{3}} \right) = \boxed{\frac{3}{2} \cdot 5^{\frac{2}{3}}}$$

$$\text{Bonus. } \int \cot^4 \theta d\theta = \int \cot^2 \theta (\csc^2 \theta - 1) d\theta$$

$$= \int \cot^2 \theta \csc^2 \theta - \cot^2 \theta d\theta = \int \cot^2 \theta \csc^2 \theta - \csc^2 \theta + 1 d\theta$$

$$= \boxed{-\frac{\cot^3 \theta}{3} + \cot \theta + \theta + C}$$