

Name: _____

Evaluate each integral.

$$1. \int \sin^2(3x) dx = \int \frac{1 - \cos(6x)}{2} dx = \int \frac{1}{2} - \frac{\cos(6x)}{2} dx$$

$$= \boxed{\frac{x}{2} - \frac{\sin(6x)}{12} + C}$$

$$2. \int_0^{\frac{\pi}{2}} \sin^2(t) \cos^5(t) dt = \int_0^{\frac{\pi}{2}} \sin^2(t) (\cos^2(t))^2 \cos(t) dt$$

$$= \int_0^{\frac{\pi}{2}} \sin^2(t) (1 - \sin^2(t))^2 \cos(t) dt = \int_0^1 u^2 (1 - u^2)^2 du$$

$$\left[\begin{array}{l} u = \sin(t) \\ du = \cos(t) dt \end{array} \right] = \int_0^1 u^2 - 2u^4 + u^6 du = \left(\frac{u^3}{3} - \frac{2u^5}{5} + \frac{u^7}{7} \right) \Big|_0^1$$

$$= \boxed{\frac{1}{3} - \frac{2}{5} + \frac{1}{7}}$$

$$3. \int \sec^3(\theta) \tan^3(\theta) d\theta$$

$$= \int \sec^2(\theta) \tan^2(\theta) \sec(\theta) \tan(\theta) d\theta$$

$$= \int \sec^2(\theta) (\sec^2(\theta) - 1) \sec(\theta) \tan(\theta) d\theta \quad \left[\begin{array}{l} u = \sec(\theta) \\ du = \sec(\theta) \tan(\theta) d\theta \end{array} \right]$$

$$= \int u^2 (u^2 - 1) du = \int u^4 - u^2 du$$

$$= \frac{u^5}{5} - \frac{u^3}{3} + C = \boxed{\frac{\sec^5(\theta)}{5} - \frac{\sec^3(\theta)}{3} + C}$$