Math 226 Sections 29 and 33 Quiz 2 (make up)

Name:_____

1. Find the limit.

$$\lim_{x\to\infty} \frac{\ln\sqrt{x}}{x^2}$$

$= \lim_{x\to\infty} \frac{\frac{1}{2}\ln x}{x^2}$ (type $\frac{\infty}{\infty}$) $\overset{l'H}{=} \lim_{x\to\infty} \frac{\left(\frac{1}{2x}\right)}{2x} = \lim_{x\to\infty} \frac{1}{4x^2} = \boxed{0}$

2. Evaluate the integral.

$$\int \frac{1}{\sqrt{1-x^2}\sin^{-1}x}\,dx$$

Let $u = \sin^{-1}x$

$du = \frac{1}{\sqrt{1-x^2}}\,dx$

$= \int \frac{1}{u}\,du$

$= \ln|u| + C$

$= \boxed{\ln|\sin^{-1}x| + C}$

3. Carbon-14 has a half-life of about 5730 years. What percent of a sample of carbon-14 will be left after 2000 years?

$m(t) = m_0 e^{kt}$

$m(5730) = m_0 e^{k\cdot 5730} = \frac{1}{2} m_0$

$e^{k\cdot 5730} = \frac{1}{2}$

$k \cdot 5730 = \ln\frac{1}{2}$

$\boxed{k = \frac{\ln\frac{1}{2}}{5730}}$

$m(2000) = m_0 e^{k\cdot 2000}$

$\boxed{\left(100\, e^{k\cdot 2000}\right)\% \text{ will remain.}}$