

Name: Solutions

1. Determine if each sequence is monotonic or not, bounded or not, and convergent or not. Write "yes" or "no" in the appropriate cell. You don't have to show any work for this problem.

	monotonic	bounded	convergent
$\frac{1}{n}$	yes	yes	yes
$\sin(n)$	no	yes	no
$3n$	yes	no	no

2. Determine whether each sequence converges or diverges. If it converges, find the limit.

(a) $a_n = ne^{-n}$

$$\lim_{n \rightarrow \infty} ne^{-n} = \lim_{n \rightarrow \infty} \frac{n}{e^n} \stackrel{\text{l'H}}{=} \lim_{n \rightarrow \infty} \frac{1}{e^n} = 0 \quad \text{convergent.}$$

type $\frac{\infty}{\infty}$

(b) $a_n = 5^n 3^{-2n}$

$$\lim_{n \rightarrow \infty} 5^n 3^{-2n} = \lim_{n \rightarrow \infty} \left(\frac{5}{3^2}\right)^n = \lim_{n \rightarrow \infty} \left(\frac{5}{9}\right)^n = 0 \quad \text{convergent.}$$

(c) $a_n = \arctan(n)$

$$\lim_{n \rightarrow \infty} \arctan(n) = \frac{\pi}{2} \quad \text{convergent.}$$

3. Use the Squeeze Theorem to show that $\left\{\frac{1}{n!}\right\}$ converges.

$$0 \leq \frac{1}{n!} \leq \frac{1}{n} \quad \text{for each } n$$

$$\lim_{n \rightarrow \infty} 0 = 0, \quad \lim_{n \rightarrow \infty} \frac{1}{n} = 0.$$

By the Squeeze Theorem, $\lim_{n \rightarrow \infty} \frac{1}{n!} = 0.$