Math 227 Sections 7 and 10 Quiz 6 10/25/17

Name: Solutions

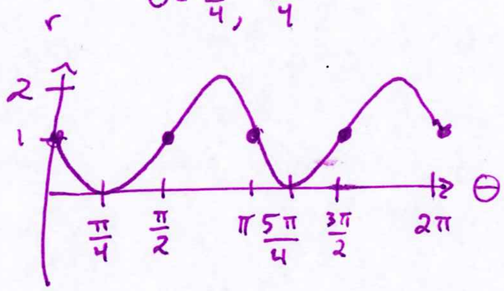
1. Sketch the curve C given by the polar equation $r = 1 - \sin(2\theta)$ by first sketching the graph of r as a function of θ in Cartesian coordinates. Label the axes of both graphs.

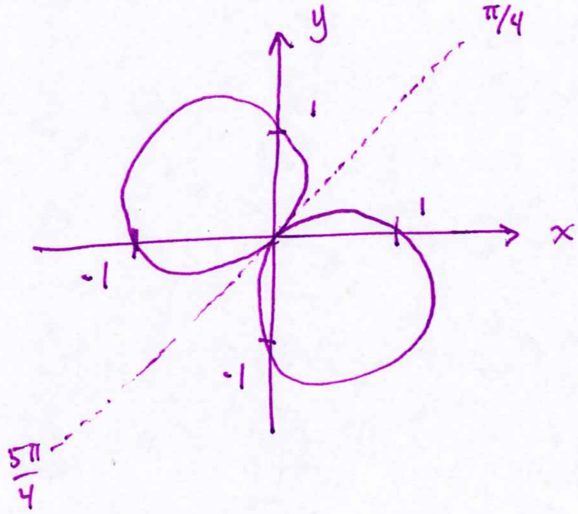
$1 - \sin(2\theta) = 0$

$\sin(2\theta) = 1$

$2\theta = \frac{\pi}{2} + 2\pi n$

$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$





2. Find the slope of the tangent line of C at $\theta = \frac{\pi}{2}$.

$$r\Big|_{\frac{\pi}{2}} = 1 - \sin(\pi) = 1 \qquad \frac{dr}{d\theta}\Big|_{\frac{\pi}{2}} = -2\cos(2\theta)\Big|_{\frac{\pi}{2}} = -2\cos(\pi) = 2$$

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{\frac{d}{d\theta}(r\sin\theta)}{\frac{d}{d\theta}(r\cos\theta)} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

$$\frac{dy}{dx}\Big|_{\theta=\frac{\pi}{2}} = \frac{\frac{dr}{d\theta}\big|_{\frac{\pi}{2}}\sin\left(\frac{\pi}{2}\right) + r\big|_{\frac{\pi}{2}}\cos\left(\frac{\pi}{2}\right)}{\frac{dr}{d\theta}\big|_{\frac{\pi}{2}}\cos\left(\frac{\pi}{2}\right) - r\big|_{\frac{\pi}{2}}\sin\left(\frac{\pi}{2}\right)} = \frac{(2)(1) + (1)(0)}{(2)(0) - (1)(1)} = \frac{2}{-1} = -2$$

3. Set up an integral that gives the area enclosed by C. You don't have to evaluate the integral.

$$\int_0^{2\pi} \frac{1}{2} r^2 \, d\theta = \int_0^{2\pi} \frac{1}{2}\left(1 - \sin(2\theta)\right)^2 d\theta$$

4. Set up an integral that gives the arc length of C. You don't have to evaluate the integral.

$$\int_0^{2\pi} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta = \int_0^{2\pi} \sqrt{(1 - \sin(2\theta))^2 + (-2\cos(2\theta))^2}\, d\theta$$