Math 226 Sections 7 and 10 Quiz 4 Solutions **9/29/17**

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1. Evaluate each integral.

(a) $\int_0^{2/3} \sqrt{4-9x^2}\,dx$

$$= \int_0^{\frac{\pi}{2}} \sqrt{4-4\sin^2\theta}\,\tfrac{2}{3}\cos\theta\,d\theta = \int_0^{\frac{\pi}{2}} \sqrt{4\cos^2\theta}\cdot\tfrac{2}{3}\cos\theta\,d\theta$$

$x = \frac{2}{3}\sin\theta$

$dx = \frac{2}{3}\cos\theta\,d\theta$

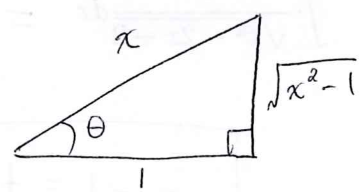
$$= \frac{4}{3}\int_0^{\frac{\pi}{2}} \cos^2\theta\,d\theta = \frac{4}{3}\int_0^{\frac{\pi}{2}} \frac{1+\cos(2\theta)}{2}\,d\theta$$

$$= \frac{4}{3}\left(\frac{\theta}{2} + \frac{\sin(2\theta)}{4}\right)\Bigg|_0^{\frac{\pi}{2}} = \frac{4}{3}\left(\frac{\pi}{4} - 0\right) = \frac{\pi}{3}$$

(b) $\int \frac{\sqrt{x^2-1}}{x^4}\,dx$

$x = \sec\theta$

$dx = \sec\theta\tan\theta\,d\theta$



$$= \int \frac{\sqrt{\sec^2\theta - 1}}{\sec^4\theta}\cdot\sec\theta\tan\theta\,d\theta = \int \frac{\sqrt{\tan^2\theta}}{\sec^3\theta}\cdot\tan\theta\,d\theta = \int \frac{\tan^2\theta}{\sec^3\theta}\,d\theta$$

$$= \int \sin^2\theta\cos\theta\,d\theta = \int u^2\,du = \frac{u^3}{3} + C = \frac{\sin^3\theta}{3} + C$$

$$= \frac{1}{3}\left(\frac{\sqrt{x^2-1}}{x}\right)^3 + C$$

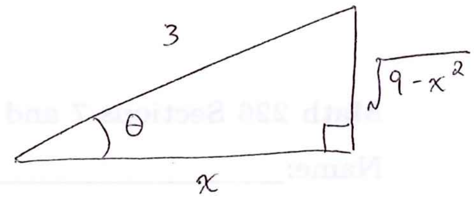
$u = \sin\theta$

$du = \cos\theta\,d\theta$

(c) $\int \frac{x^2}{\sqrt{9-x^2}}dx$

$x = 3\sin\theta$

$dx = 3\cos\theta \, d\theta$



$$= \int \frac{9\sin^2\theta}{\sqrt{9-9\sin^2\theta}} 3\cos\theta \, d\theta = \int \frac{9\sin^2\theta}{\sqrt{9\cos^2\theta}} \cdot 3\cos\theta \, d\theta = 9\int \sin^2\theta \, d\theta$$

$$= 9\int \frac{1-\cos(2\theta)}{2} d\theta = 9\left(\frac{\theta}{2} - \frac{\sin(2\theta)}{4}\right) + C$$

$$= 9\left(\frac{\theta}{2} - \frac{\sin(\theta)\cos(\theta)}{2}\right) + C = \frac{9}{2}\left(\theta - \sin(\theta)\cos(\theta)\right) + C$$

$$= \boxed{\frac{9}{2}\left(\arcsin\left(\frac{x}{3}\right) - \left(\frac{x}{3}\right)\left(\frac{\sqrt{9-x^2}}{3}\right)\right) + C}$$

2. Find the correct trigonometric substitution (in other words, express x and dx in terms of θ and $d\theta$) and draw the corresponding triangle. You do not have to evaluate the integral.

$$\int \frac{x^2}{\sqrt{x^2-2x+2}}dx = \int \frac{x^2}{\sqrt{(x-1)^2+1}} dx$$

$x - 1 = \tan\theta$

$dx = \sec^2\theta \, d\theta$