Math 220-04 Integration Quiz #2/3 Sol^n

Always check to see if you can integrate with a u-sub first. It could be as simple as u du or $u^n du$. #1 below is the second of these.

Or it could be a ln u du or $e^u du$. If the entire integrand doesn't get 'taken up' by the substitutions you chose, then IBP is probably the way to go.

1. $\int_0^2 x\sqrt{4-x^2}\,dx$

$\int_0^2 x\sqrt{4-x^2}\,dx$

let $u = 4-x^2$ when $x=0, u=4$

$du = -2x\,dx$ $x=2, u=0$

$x\,dx = -\frac{1}{2}du$

$= -\frac{1}{2}\int_4^0 \sqrt{u}\,du = \frac{1}{2}\int_0^4 u^{\frac{1}{2}}\,du$

$= \frac{1}{1+\frac{1}{2}}u^{\frac{1}{2}+1}\Big|_0^4 = \frac{2}{3}(4)^{\frac{3}{2}} - \frac{2}{3}(0) = \frac{16}{3}$

2. $\int x^2 \ln x\,dx$

let $u = \ln x$ $= \frac{1}{3}x^3\ln x - \int \frac{1}{3}x^3 \cdot \frac{1}{x}\,dx$

$dv = x^2\,dx$ $= \frac{1}{3}x^3\ln x - \frac{1}{3}\int x^2\,dx$

$v = \frac{1}{3}x^3$

$du = \frac{1}{x}\,dx$ $= \frac{1}{3}x^3\ln x - \frac{1}{9}x^3 + C$

3. Recalling, as integrating by parts shows, that $\int \ln x\,dx = x\ln x - x + C$, what is the average value of $\ln x$ on $[1, e^2]$?

Avg value of $\ln x$ on $[1, e^2]$ is

$$\frac{1}{e^2-1}\int_1^{e^2} \ln x\,dx$$

$$= \frac{1}{e^2-1}(x\ln x - x)\Big|_1^{e^2} = \frac{1}{e^2-1}\left(e^2\ln(e^2) - e^2 - 1\ln(1) + 1\right)$$

$$= \frac{1}{e^2-1}(e^2+1) = \frac{e^2+1}{e^2-1}$$

4. Find the area between the graphs of $f(x) = -x^2 + 10$ and $g(x) = x + 4$.

[*Hint*: To find the limits of integration, solve where the functions intersect.]

$f(x) = g(x) \Rightarrow -x^2 + 10 = x + 4 \Rightarrow x^2 + x - 6 = 0 \Rightarrow x = 2$ or $x = -3$.

pick $x = 0$, $g(0) = 4 < 10 = f(0)$

$$\therefore \text{Area} = \int_{-3}^{2} f(x) - g(x)\, dx$$

$$= \int_{-3}^{2} -x^2 - x + 6\, dx = -\frac{1}{3}x^3 - \frac{1}{2}x^2 + 6x \Big|_{-3}^{2}$$

$$= -\frac{1}{3}(2)^3 - \frac{1}{2}(2)^2 + 12 + \frac{1}{3}(-3)^3 + \frac{1}{2}(-3)^2 - 6(-3) = 20\frac{5}{6}.$$

5. A faucet is turned on and water flows out at a rate of $V(t) = t^3 - \frac{1}{2}t^2 + 4$ gallons per minute, where t is the number of minutes since the faucet was turned on.

To the nearest gallon, how much water flows out of the faucet during the first two minutes the faucet is turned on?

Total Amount of water $= \int_0^2 V(t)\, dt = \int_0^2 t^3 - \frac{1}{2}t^2 + 4\, dt$

$$= \frac{1}{4}t^4 - \frac{1}{6}t^3 + 4t \Big|_0^2$$

$$= \frac{1}{4} \cdot 16 - \frac{1}{6} \cdot 8 + 8$$

$$= 4 - 2 + 8 = 10 \text{ gallons}.$$

6. Suppose the marginal profit from the sale of x hundred items is $P'(x) = 4 - 6x + 9x^2$, and the profit on 0 items is −\$60. Find the profit function.

profit $P(x) = \int P'(x)\, dx = \int 4 - 6x + 9x^2\, dx$

$$= 4x - 3x^2 + 3x^3 + C.$$

$\therefore P(x) = 4x - 3x^2 + 3x^3 - 60$

When $x = 0$, $P(x) = -60 \Rightarrow P(0) = 0 - 0 + 0 + C = -60 \Rightarrow C = -60$