

220 Review

Chapter 1 - 11

Functions

variable, formula.
(independent variable)

values

* functions are changing as variable changes.
* identify variable carefully.

* Think about the following 5 properties when you get a fcn:
(Ask yourself the following 5 questions)

1. Domain: all variable values s.t. your fcn makes sense.

e.g. $f(x)$ is a fcn of x
↑
domain D_x is the set of x s.t. $f(x)$ is well defined.

2. Range: all possible fcn values.

3. What's the graph of the fcn?

* NOTE: If the fcn you are given has a graph which you know it and is easy to sketch, sketch the graph first!

e.g. linear fcn are straight lines.

quadratic fcn are parabolas.

→ x-intercept: where your fcn cross the x-axis on an x-y-plane. let $f(x)=0$, solve for x.

→ y-intercept: where your fcn cross the y-axis. let $x=0$, (plug $x=0$ into fcn)

* Continuity

Defⁿ A fcn is continuous @ $x=a$ means. $\lim_{x \rightarrow a} f(x) = f(a)$.

Defⁿ A fcn is called a continuous fcn if and only if it is continuous @ each point on its domain.

* NOTE: piecewise fcn are the most interesting group of fcn who might have discontinuous pts in economics.

When given a piecewise defined fcn, look @ the endpoints of each piece of its domain. discontinuous point might be among those endpoints.

* Limit: how your fun will behave when variable get closed enough to some point.

limit exists iff and only if $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = \text{some } \#$ ($< \infty$)

* Some notes on limits Suppose $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ exist.

• $\lim_{x \rightarrow a} (f(x) \pm g(x)) = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x)$

• $\lim_{x \rightarrow a} \text{Constant} = \text{Constant}$.

• $\lim_{x \rightarrow a} c(f(x)) = c \lim_{x \rightarrow a} f(x)$, c is a constant.

• $\lim_{x \rightarrow a} (f(x) \cdot g(x)) = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$

• $\lim_{x \rightarrow a} \left(\frac{f(x)}{g(x)} \right) = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$, given $\lim_{x \rightarrow a} g(x) \neq 0$

! If $\lim_{x \rightarrow a} g(x) = 0$, simplify $\frac{f(x)}{g(x)}$ first, until the limit of denominator is nonzero.

5. Derivatives

Defⁿ $f'(x) := \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

Interpretation: * derivative of a fun @ $x=a$

is the slope of the tangent line @ $x=a$.

* derivative of a fun is also the instantaneous rate of change ~~of the function~~.

* Special case in economic terms: ~~deriv~~

when talking about "marginal", take derivatives.

* Calculate derivatives: ~~deriv~~

• $(x^n)' = n \cdot x^{n-1}$, $n \in \mathbb{R}$.

• $(\text{constant})' = 0$

• $(e^x)' = e^x$

• $(\ln x)' = \frac{1}{x}$.

→ Suppose $f'(x)$, $g'(x)$ exist,

• $(f(x) \pm g(x))' = f'(x) \pm g'(x)$

$$\bullet (c \cdot f(x))' = c \cdot f'(x), \quad c \text{ is constant real \#}$$

$$\bullet (f(x) \cdot g(x))' = f'(x)g(x) + f(x)g'(x)$$

Product Rule

$$\bullet \left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$$

Quotient Rule

★ NOTE on quotient rule

When you get a quotient form fun. before applying quotient rule ~~etc~~ directly, try to simplify your fun first!

$$\text{e.g. } \frac{1}{x} = x^{-1}, \quad \frac{1}{x^2} = x^{-2}$$

$$\frac{x^2 - 4x + 5}{x - 5} = \frac{(x+1)(x-5)}{x-5} = x+1, \quad \text{etc.}$$

$$\bullet (f(g(x)))' = f'(g(x)) \cdot g'(x)$$

Chain Rule.

Special Funs we've learned

1. Quadratic funs: $f(x) = ax^2 + bx + c$

* Parabolic graphs.

* Could be factored as product of two linear funs.

at most
* two roots (x-intercepts) when $b^2 \geq 4ac$, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

* Straight line motions: if velocity is a linear fun in terms of time, then position is a quadratic fun.

2. Linear funs: #

* many different forms.

* Very common in economics!

• Revenue = unit price (p) \times # of units ~~of~~ of sale (x) = px .

• Cost is often a linear fun of # of unit produced.

POLYNOMIALS

• profit = Revenue - Cost.
= 0 when the business break even.

• ~~Revenue~~ demand: the # of units sold.

Sometimes it is a linear fn of unit price.
given as

* Graph of linear fns are straight lines.

hence derivatives are the slopes of those straight lines.

3. Exponential fns and Logarithmic fns.

* Check your exponential and logarithmic fn
Formula sheets !

* understand ~~that~~ the connection between exponential
fns and interest rate.

(See also the exponential and log fn formulae.)