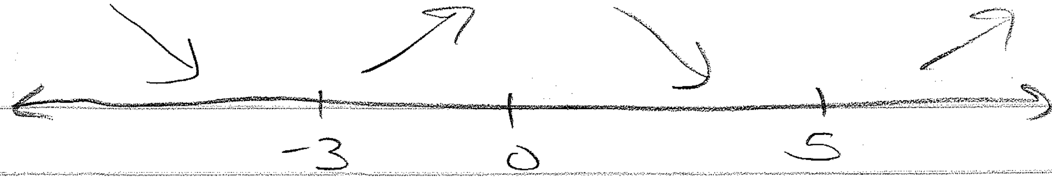


Test $x = -4, -1, 1, 6$

(Substitute these into the factored derivative.

$f'(-4) = (-)(-)(-) = (-) < 0 \quad f \downarrow$

$f'(-1) = (-)(-)(+) = (+) > 0 \quad f \uparrow$

$f'(1) = (+)(-)(+) = (-) < 0 \quad f \downarrow$

$f'(6) = (+)(+)(+) = (+) > 0 \quad f \uparrow$

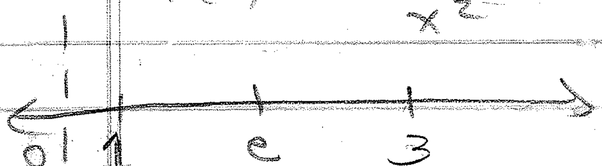
f increasing on $(-3, 0) \cup (5, \infty)$

f decreasing on $(-\infty, -3) \cup (0, 5)$

#2l) $f(x) = \dfrac{\ln x}{x}$ Dom!! $x > 0$ since $\ln x$ is the determining part of the fun. since it's most restricting.

$f'(x) = \dfrac{(1/x)(x) - (\ln x)(1)}{x^2}$

$f'(x) = \dfrac{1 - \ln x}{x^2} = 0$ at $\ln x = 1$ or $\boxed{x = e}$ & DNE at $x = 0$, but that's not in domain, so discard it. (There is an asymptote at $x = 0$)



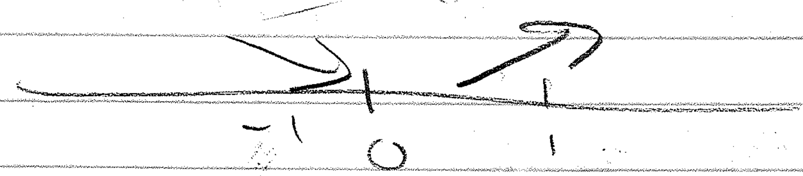
Test $x = 1, 3$

$f'(1) = \dfrac{1 - \ln 1}{1^2} = 1 - 0 > 0$, $f \uparrow$ on $(0, e)$

$f'(3) = \dfrac{1 - \ln 3}{3^2} < 0$ (Why?), $f \downarrow$ on (e, ∞)

m) $f(x) = x - e^x$ Domain $\mathbb{R}$

$f'(x) = 1 - e^x = 0 \to 1 = e^x \to x = 0$



$f'(-1) = 1 - e^{-1} = 1 - \frac{1}{e} > 0$

$f'(1) = 1 - e^{1} < 0$

So $f(0)$ must be a local max.

f increases on $(-\infty, 0)$ decreases $(0, \infty)$

$f''(x) < 0$

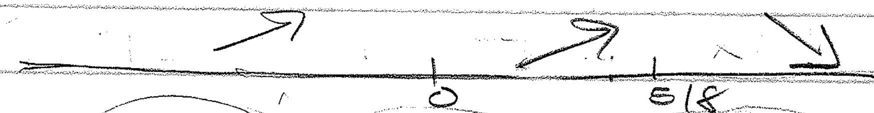
f is decreasing

$x=0$ is indeed a critical value, but not an extreme val

#2 i) $y = x^{5/3} - x^{8/3}$

$y' = \frac{5}{3}x^{2/3} - \frac{8}{3}x^{5/3} = \frac{1}{3}x^{2/3}(5-8x) = 0$

$y' = 0$ when $x = 0$ or $x = 5/8$



Expression to test values in $\boxed{\frac{x^{2/3}}{3}(5-8x)}$

Test $x=-1$ $x=\frac{1}{2}$ $x=1$

$f'(-1) = (+)(+) > 0$, $f \uparrow$ on $(-\infty, 0)$

$f'(1/2) = (+)(+) > 0$, $f \uparrow$ on $(0, 5/8)$

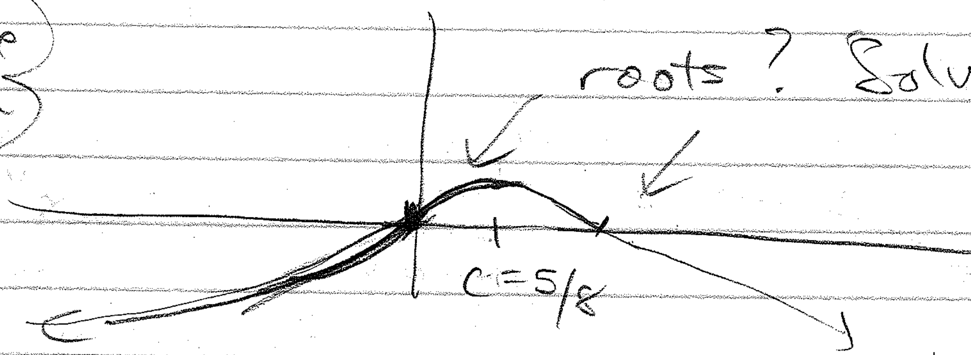
$f'(1) = (+)(-) < 0$, $f \downarrow$ on $(5/8, \infty)$

✓ $x=0$ is a crit value but not an extreme

✓ $x = 5/8$ is a local max

Possible sketch

roots? Solve $f(x)=0$:



$f(x) = x^{5/3} - x^{8/3} = x^{5/3}(1-x) = 0$ at $\boxed{x=0,1}$

#2 j) $f(x) = 3x^4 - 8x^3 - 90x^2 + 70$

$f'(x) = 12x^3 - 24x^2 - 180x = 12x(x^2-2x-15)$

Factor: $f'(x) = 12x(x-5)(x+3) = 0$ at $x = 0, 5, -3$

Since $f(x)$ is a polynomial, all these are in domain

#2f) Con'd $f(-\sqrt{3}) = -\sqrt{3} + \frac{3}{-\sqrt{3}} = \frac{+3-3}{\sqrt{3}} = 0$

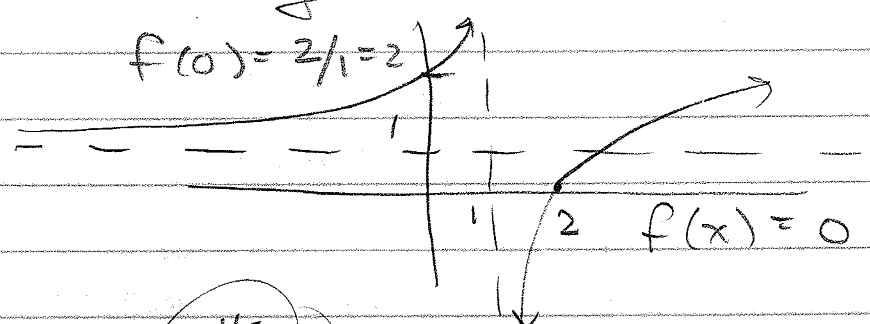
$f(\sqrt{3}) = \sqrt{3} + \frac{3}{\sqrt{3}} = \frac{3+3}{\sqrt{3}} = \frac{6}{\sqrt{3}} = 2\sqrt{3}$

So, $x = -\sqrt{3}$ is a root of f.

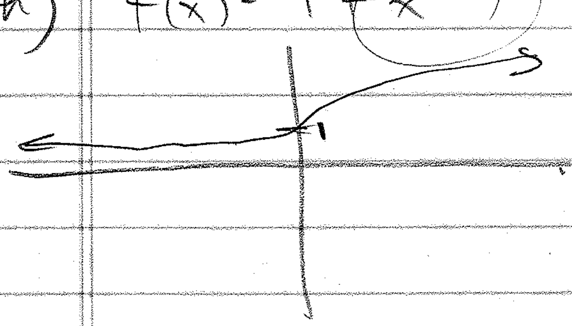
#2g) $f(x) = \frac{x-2}{x-1}$, $f'(x) = \frac{(x-1)-(x-2)}{(x-1)^2} = \frac{1}{(x-1)^2}$

$f'(x)$ is $>$ zero for all x, since $(x-1)^2 > 0$ for all x.

- The fcn. DNE at $x=1$, so $x=1$ is not a crit #. However, it is an asymptote.
- f is increasing on $(-\infty, 0) \cup (0, \infty)$



#2h) $f(x) = 1 + x^{1/5}$ $\longrightarrow$ We know this is an odd root fcn, so it's like $x^{1/3}$, but it's moved up 1 and it's flatter. Expect f' to increase on all the reals (which is the domain)



$f'(x) = \frac{1}{5}x^{-4/5} = \frac{1}{5x^{4/5}}$ which DNE at $x=0$

Where is $\frac{1}{5x^{4/5}} > 0$? Where $x^{4/5} > 0$.

Write $x^{4/5}$ as $(x^{1/5})^4 = (+/-)^4 = +$

So f is increasing on $(-\infty, 0) \cup (0, \infty)$

e) $y = -2x + 4$, $y' = -2 < 0$ for all x

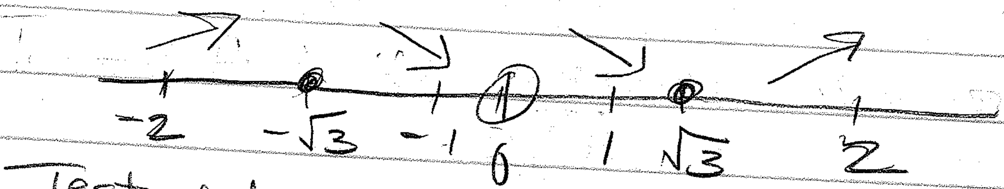
So f decreasing on $\mathbb{R}$, no extremes

f) $\boxed{f(x) = x + \frac{3}{x}}$, $f'(x) = 1 - \frac{3}{x^2} = \frac{x^2 - 3}{x^2}$

Dom: $x \neq 0$

$f' = \frac{x^2 - 3}{x^2} = 0$ at $x = \pm\sqrt{3}$ + DNE at $x = 0$

However, $f(0)$ also DNE, so $x = 0$ is not crit. valu



Test values: $x = -2, -1, 1, 2$

$f'(-2) = \frac{4-3}{4} > 0$, $f'(-1) = \frac{1-3}{1} < 0$,

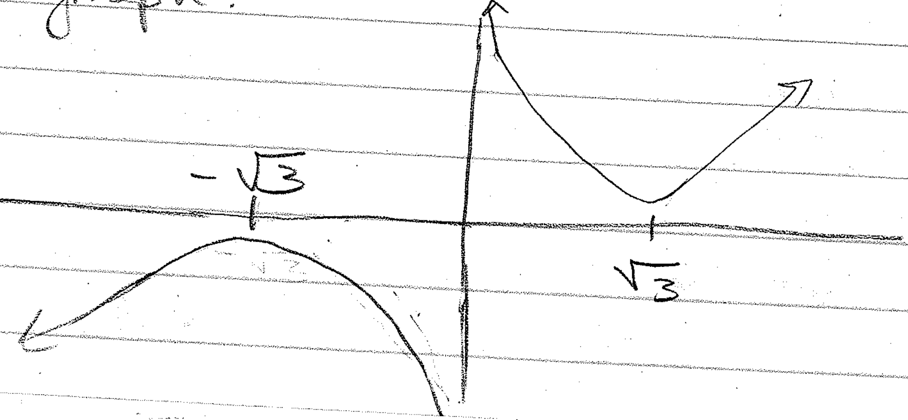
$f'(1) = \frac{1-3}{1} < 0$, $f'(2) = \frac{4-3}{4} > 0$.

So, local min at $x = \sqrt{3}$, local max at $x = -\sqrt{3}$

f increasing on $(-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty)$

f decreasing on $(-\sqrt{3}, 0) \cup (0, \sqrt{3})$

To begin sketching the curve of $f(x) = x + \frac{3}{x}$ we look at the data just found and apply it to the graph:



What is the value of $f(x)$ at $x = \pm\sqrt{3}$?

#2c) $f(x) = \frac{2}{3}x^3 - x^2 - 24x - 10$

$f'(x) = 2x^2 - 2x - 24 = 2(x^2 - x - 12)$

$= 2(x-4)(x+3) = 0$ when $x = 4$ or -3

test $\quad$ test $\quad$ test

$-4 \quad -3 \quad 0 \quad 4 \quad 5$

$f'(-4) = 16 + 4 - 12 > 0$ $\}$ $f(-3)$ local max

$f'(0) = -12 < 0$

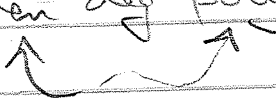
$f'(5) = 25 - 5 - 12 > 0$ $\}$ $f(4)$ local min

Increasing on $(-\infty, -3) \cup (4, \infty)$

Decreasing on $(-3, 4)$

d) $f(x) = (x^2 - 1)^8 \qquad f'(x) = 8(x^2-1)(2x) = 0$

$\hookrightarrow$ Even deg poly $\qquad$ at $x = 1, -1, 0$



$-2 \quad -1 \quad -\frac{1}{2} \quad 0 \quad \frac{1}{2} \quad 1 \quad 2$

test pts $\longrightarrow$

$f'(-2) = (+)(-) < 0$ $\}$ $f(-1)$ local min

$f'(-\frac{1}{2}) = (-)(-) > 0$ $\}$ $f(0)$ local max

$f'(\frac{1}{2}) = (-)(+) < 0$

$f'(2) = (+)(+) > 0$ $\}$ $f(1)$ local min

Increasing $(-1, 0) \cup (1, \infty)$

Decreasing $(-\infty, -1) \cup (0, 1)$

My work is a little disorganized. You should lay out your number line with c values clearly marked and do your test pt work below it. Then draw your increasing or decreasing arrows on the number line. Finally, write the interval notation of your answer and state your local extremes at "local __ at $x =$ __

In #1, $x = -3, -1, 1$, and 2 appear to be crit. numbers.

<u>Notice</u>: $f'(-3) = 0$, $f'(-1) = 0$ & $f'(2) = 0$

$f(-3)$ local min $\quad f(-1)$ + $f(2)$ inflection

There's a cusp at $x = 1$

#2. <u>First der. test</u> - If c is a crit value of f and the sign of f' changes from positive to negative at c (i.e., $f'(x) < 0$ for $x < c$ and $f'(x) > 0$ for $x > c$), then $f(c)$ is a local max. (Be able to write this on a quiz or test)

a) $y' = 24 - 16x = 8(3 - 2x) = 0$ at $x = \frac{3}{2}$

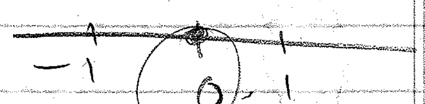
$y'\big|_{x=1} > 0$ and $y'\big|_{x=2} < 0$, therefore $f(3/2)$ is a local max.

b) $f(x) = x^4 - 5x^3 + 100$

$f'(x) = 4x^3 - 15x^2 = x^2(4x - 15) = 0$

at $x = 15/4$.

$\left.\begin{array}{l} f'(3) < 0 \\ f'(4) > 0 \end{array}\right\}$ $f(3^3/4)$ is a local min



$\left.\begin{array}{l} f'(-1) < 0 \\ f'(1) < 0 \end{array}\right\}$ $f(0)$ is pt of inflection

Be sure to write the <u>intervals</u> where f is increasing + decreasing

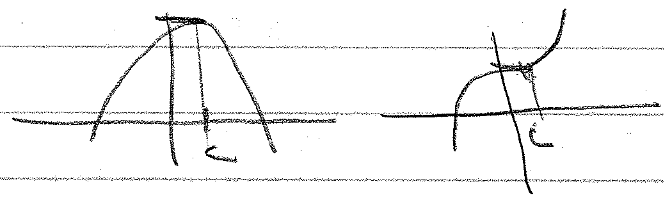
Sec 17 HW #13

#1. Critical values are those $x=c$ in domain of f where:

1. $f'(c)=0$

or 2. $f'(c)$ DNE

If $f'(c)=0$, then c is either where a local extreme occurs or where there is an inflection pt.



If $f'(c)$ DNE, either $f(c)$ is a cusp or corner or it's a vertical tangent.

NOTE: $f(c)$ DNE, then $f'(c)$ DNE does not lead to concluding that c is a critical value.

Ex $f(x)=\sqrt{x}$ Dom: $x \geq 0$

$f'(x) = \frac{1}{2\sqrt{x}}$ which DNE at $x=0$

So $x=0$ is a crit. value.

Ex $f(x) = \frac{1}{x}$ Dom: $x \neq 0$

$f'(x) = \frac{-1}{x^2}$ DNE at $x=0$ but neither does $f(0)$, so $x=0$ is not crit #