Essential Derivative Forms for Business and Economics Applications

These (except the first) relate to the *power rule*:

$f(x) = c$ $\quad$ $f'(x) = 0$ $\quad$ (c is a constant)

$f(x) = cx$ $\quad$ $f'(x) = c$

$f(x) = x^n$ $\quad$ $f'(x) = nx^{n-1}$ $\quad$ (n is a real number $\neq$ zero)

$f(x) = cx^n$ $\quad$ $f'(x) = ncx^{n-1}$

$f(x) = \sqrt{x}$ $\quad$ $f'(x) = \dfrac{1}{2\sqrt{x}}$

The next are the *exponential and logarithmic derivatives* (natural and general):

$f(x) = e^x$ $\quad$ $f'(x) = e^x$

$f(x) = a^x$ $\quad$ $f'(x) = a^x \ln a$

$f(x) = \ln x$ $\quad$ $f'(x) = \dfrac{1}{x}$

$f(x) = \log_a x$ $\quad$ $f'(x) = \dfrac{1}{x \ln a}$

The *properties of derivatives for sums and differences* as well as for constant follow (they are easy to verify):

$[f(x) \pm g(x)]' = f'(x) \pm g'(x)$ $\quad$ $[f(cx)]' = cf'(x)$

Hence, the derivative of a polynomial is the sum of the monomials that it comprises.

$f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x^1 + a_0$ $\quad$ $f'(x) = na_n x^{n-1} + (n-1)a_{n-1}x^{n-2} + \ldots + a_1 + 0$

The *chain rule* for differentiation applies when f is *composed* with another function, u.

$$\left[(f \circ u)(x)\right]' \equiv \left[f(u(x))\right]' = f'(u(x))u'(x)$$

The chain rule is employed in the *u-substitution forms*, where f is a function of u, where u is a function of x. They are analogous to the first set:

$$f(u(x)) = \left[u(x)\right]^n \qquad f'(u(x)) = n(u(x))^{n-1}u'(x)$$

$$f(u(x)) = \sqrt{u(x)} \qquad f'(u(x)) = \frac{1}{2\sqrt{u(x)}}u'(x)$$

$$f(u(x)) = e^{u(x)} \qquad f'(u(x)) = e^{u(x)}u'(x)$$

$$f(x) = a^{u(x)} \qquad f'(x) = a^{u(x)}u'(x)\ln a$$

$$f(x) = \ln u(x) \qquad f'(x) = \frac{1}{u(x)}u'(x)$$

$$f(x) = \log_a u(x) \qquad f'(x) = \frac{1}{u(x)\ln a}u'(x)$$

The properties don't hold for products and quotients of derivatives. For these we need the *product and quotient rules*:

$$\left[f(x)g(x)\right]' = f'(x)g(x) + g'(x)f(x) \qquad \left[\frac{f(x)}{g(x)}\right]' = \frac{f'(x)g(x) - g'(x)f(x)}{\left[g(x)\right]^2}$$

The analogous u-substitution forms are as you would expect. It's best to do examples and let them develop organically than to formulate.