

Quiz 9

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1. For a certain product, it is known that the relationship between price (p) and demand (q) is given as $p = -0.02q + 12$.

- (a) Give the expression of revenue as a function of demand q . At which demand level will the revenue get maximized?

$$R(q) = pq = q(-0.02q + 12)$$

$$R'(q) = -0.02 \times 2q + 12$$

$$R''(q) = -0.04 < 0 \quad = 300$$

- (b) Find the elasticity function $E(p)$ (as a function of price p). \therefore when $q=300$ revenue maxed.

$$q = \frac{12-p}{0.02} = -50p + 600 \quad \frac{dq}{dp} = -50$$

$$E(p) = -\frac{p}{q} \cdot \frac{dq}{dp} = \frac{p}{-50p+600} \cdot (-50) = \frac{50p}{-50p+600} = \frac{-p}{p-12}$$

- i. Calculate $E(10)$ and interpret it.

$$E(10) = \frac{-10}{10-12} = 5 > 1$$

- \therefore when $p=10$ demand is elastic hence when there's a small increase in price the revenue will decrease

- ii. At which price level will the revenue get maximized?

$$\text{When } E(p) = 1 \Rightarrow \frac{-p}{p-12} = 1 \Leftrightarrow -p = p-12 \Rightarrow \boxed{p=6}$$

2. Consider the function: $f(x, y) = x \ln(xy)$

- (a) What's the domain of $f(x, y)$?

$$D_{(x,y)} = \{(x, y) : xy > 0\} \text{ or } \{(x, y) : x > 0, y > 0 \text{ or } x < 0, y < 0\}$$

- (b) Calculate the first order partial derivatives $f_x(x, y)$ and $f_y(x, y)$:

$$f_x(x, y) = \frac{xy}{xy} + \ln(xy) = 1 + \ln(xy)$$

$$f_y(x, y) = x \cdot \frac{1}{xy} \cdot x = \frac{x}{y}$$

- (c) Calculate the second order partial derivative $f_{xy}(x, y)$:

$$f_{xy}(x, y) = \frac{\partial}{\partial y} (f_x(x, y)) = x \cdot \frac{\partial}{\partial y} \left(\frac{1}{xy} \right) = \frac{1}{y}$$