

This document is intended to give some very rough guidelines for writing proofs. The first section gives some ideas approaching a statement for the first time. The second section will help you write your proof in a formal way.

## Method Guide

As you have noticed, there are quite a few proof techniques; furthermore, any given proof might require more than one technique. So how do we decide which method(s) to use when making a proof? **PRACTICE!**

One way to get hold on a statement is to write out the predicate logic translation; you can get a lot of insight from these statements once you've written them out and learned to read them well (again, PRACTICE). The most important advice I can give: DON'T BE AFRAID TO MAKE MISTAKES: JUST TRY SOMETHING. Even if your first idea doesn't work, you will probably learn something from the attempt...

In addition to my wise words above, here is a short list of vagueries that might help you get started.

- Write out the predicate logic translation of the statement you are trying to prove.  
What is the basic logical structure of the statement? What type of quantifiers?
- Write out any definitions or theorems you might need.
- Write out any assumptions you make.
- When trying to prove an implication  $P \implies Q$ :
  - Can you prove it directly by assuming  $P$  and deriving  $Q$ ?
  - Is the contrapositive  $(\neg Q) \implies (\neg P)$  easier to prove directly?
  - Can you prove it from some sequence of implications?
- When trying to prove a biconditional  $P \iff Q$ :
  - Can you prove it with a string of equivalent statements?
  - Can you prove  $P \implies Q$  and  $Q \implies P$ ?
- When trying to prove any statement:
  - Can you identify some cases to split the statement into easier statements?
  - Would it be useful to assume the contrary and derive a contradiction from that?

One final word of caution is in order: there is no general method to write every proof...

## Style Guide

The previous section glimpsed some approaches to proving a statement. This section expounds on writing proofs.

First and foremost, your proofs should be in English. A page of calculation is NOT a proof; full sentences are imperative. When writing these sentences, be mindful of the meanings of the words; if a word has a technical meaning, only use it with that meaning (e.g. the word "statement" has a formal definition in this course). Be sure that your sentences are unambiguous and actually express exactly what you are thinking.

Do not overcomplicate the proof; straightforward is better! When making a proof by contradiction, be sure you need to use proof by contradiction; if you can rephrase your proof without it, you should. Use notation when necessary, but don't get too carried away: it is often hard to read notation, and this will make your reader sad...

Here are a few suggestions (read: commands) to summarize the above.

- After discovering a proof, rewrite it on a clean sheet of paper separate from your scrap work.
- Be sure you are correctly applying theorems and technical definitions.
- Make sure that everything you do is necessary to the proof.
- Write in full sentences.
- State all assumptions, and carefully reference all propositions you use from class.
- Clearly signal to the reader what you are trying to prove and how.
- Reread what you've written to cut out any unnecessary words and notation; rewrite and check again.

A fun game: ask yourself "could a computer follow my proof, or would it throw a `NonsenseError`?"

Writing is a process; you will need to make several drafts of a proof before you can call it the finished product.