The questions below are intended for practice only. It is your responsibility to study all material covered in this course, whether represented here or not.

1. Be able to state and use any named propositions and definitions.

2. Consider the propositional statement $((P \to Q) \leftrightarrow R) \wedge (R \vee (Q \oplus (\neg P)))$.
   (a) Build a truth table for the statement.
   (b) Write the statement in Disjunctive Normal Form.
   (c) Write the statement in Conjunctive Normal Form.

3. Let $a, b, c, d \in \mathbb{Z}$ and let $m \in \mathbb{Z}_{>0}$.
   (a) Prove that if $a \mid c$ and $b \mid d$, then $ab \mid cd$.
   (b) Prove that if $a \mid b$ and $a \mid c$, then $a \mid bs + ct$ for all $s, t \in \mathbb{Z}$.

4. This question concerns equivalence relations.
   (a) Let $f \colon S \to T$ be an arbitrary function. Is $R = \{(a, b) \in S \times S : f(a) = f(b)\}$ an equivalence relation on S? Prove or disprove.
   (b) Is $R = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : x^2 + y^2 = 0\}$ an equivalence relation? Prove or disprove.
   (c) Is $R = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : x^2 = y\}$ an equivalence relation? Prove or disprove.
   (d) Is $R = \{(x, y) : xy \geq 0\}$ an equivalence relation on $\mathbb{Z} \setminus \{0\}$? Prove or disprove.

5. Let $f \colon A \to B$ and $g \colon B \to C$ be functions.
   (a) Prove that if $g \circ f$ is injective, then f is injective.
   (b) Prove that if $g \circ f$ is surjective, then g is surjective.
   (c) Give an example of functions f and g as above with $g \circ f$ a bijection, but neither f nor g is a bijection (a clear picture is an acceptable answer).

6. This question concerns induction.
   (a) Prove $\displaystyle\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$ for all $n \geq 1$.
   (b) Prove $\displaystyle\sum_{k=0}^{n} 2F_{3k+3} = F_{3n+5} - 1$ for all $n \geq 0$ where F_k is the k^{th} Fibonacci number.
   (c) Prove $\displaystyle\sum_{k=1}^{n} F_k^2 = F_n F_{n+1}$ for all $n \geq 1$ where F_k is the k^{th} Fibonacci number.

7. Let $A_0, A_1, \cdots, A_n$ be sets and $f_i \colon A_{i-1} \to A_i$ a bijection for all $1 \leq i \leq n$. Prove that $f_n \circ f_{n-1} \circ \cdots \circ f_1$ is also bijective.

8. Solve $250x \equiv 93 \pmod{927}$ for an integer x with $0 \leq x \leq 927$.

9. This question concerns the RSA Cryptosystem. Let $p = 13$, $q = 17$, and $e = 19$.
   (a) Encrypt the message $m = 15$.
   (b) Decrypt the message $\widehat{m} = 7$.