Notes on Counting Anagrams

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Definition. An *anagram* is any rearrangement of the symbols of a *word*.

Example 1. The word ODD has the following anagrams; there are three total.

1. ODD
2. DOD
3. DOD

We want to count the anagrams of a given word in general.

Remark. The answer is not $n!$ for n-letter words in general because some symbols are indistinguishable; in the example above, we cannot tell the two D's apart.

Example 2. Count the anagrams of SUPERCALIFRAGILISTICEXPIALIDOCIOUS.

Solution: This word has 34 letters total. Separating the word into like symbols we see

AAA CCC D EE F G IIIIIII LLL OO PP RR SSS T UU X.

Finally, build the anagrams of this word by placing letters of the same type simultaneously in the word (i.e. in the 34 available positions for letters) via the following procedure.

1. Place all A's. $\binom{34}{4}$ choices
2. Place all C's. $\binom{31}{3}$ choices
3. Place all D's. $\binom{28}{1}$ choices
4. Place all E's. $\binom{27}{2}$ choices
5. Place all F's. $\binom{25}{1}$ choices
6. Place all G's. $\binom{24}{1}$ choices
7. Place all I's. $\binom{23}{7}$ choices
8. Place all L's. $\binom{16}{3}$ choices
9. Place all O's. $\binom{13}{2}$ choices
10. Place all P's. $\binom{11}{2}$ choices
11. Place all R's. $\binom{9}{2}$ choices
12. Place all S's. $\binom{7}{3}$ choices
13. Place all T's. $\binom{4}{1}$ choices
14. Place all U's. $\binom{3}{2}$ choices
15. Place all X's. 1 choices

Hence by the Product Principle we obtain the precise number of anagrams below.

$$\binom{34}{4}\binom{31}{3}\binom{28}{1}\binom{27}{2}\binom{25}{1}\binom{24}{1}\binom{23}{7}\binom{16}{3}\binom{13}{2}\binom{11}{2}\binom{9}{2}\binom{7}{3}\binom{4}{1}\binom{3}{2}$$
$$=1094663885174837838366105600000 0$$

For reference, 10946638851748378383661056000000 is astronomically large (roughly one-hundred times the radius of the observable universe in millimeters). Writing one-billion anagrams every second, it would still take 25000 times the age of the universe to list them. I won't extend the homework deadline that long...