Homework 2

Instructions: Complete each of the following on separate, stapled sheets of paper.

1. Verify the given family is a solution to the indicated ODE and solve the IVP with the given ICs.
   (a) $y' = y - y^2$ has solution family $y = \dfrac{1}{1+ce^{-x}}$.
      i. $y(0) = -\frac{1}{3}$
      ii. $y(-1) = 2$
   (b) $y' + 2xy^2 = 0$ has solution family $y = \dfrac{1}{x^2+c}$.
      i. $y(2) = \frac{1}{3}$
      ii. $y(0) = 1$
   (c) $x'' + x = 0$ has solution family $x = c_1\cos(t) + c_2\sin(t)$.
      i. $x(0) = -1,\ x'(0) = 8$
      ii. $x(\frac{\pi}{6}) = \frac{1}{2},\ x'(\frac{\pi}{6}) = 0$
   (d) $x'' - x = 0$ has solution family $x = c_1e^t + c_2e^{-t}$.
      i. $x(1) = 0,\ x'(1) = e$
      ii. $x(0) = 0,\ x'(0) = 0$

2. Determine regions of the xy-plane for which the ODEs below have unique solutions through (x_0, y_0).
   (a) $y' = y^{2/3}$
   (b) $y' = \sqrt{xy}$
   (c) $xy' = y$
   (d) $y' - y = x$
   (e) $(4-y^2)y' = x^2$
   (f) $(1+y^3)y' = x^2$
   (g) $(x^2+y^2)y' = y^2$
   (h) $(y-x)y' = x+y$

3. Consider the ODE $y' = \sqrt{y^2-9}$. Are we guaranteed a unique solution through the points below?
   (a) $(1,4)$
   (b) $(5,3)$
   (c) $(2,-3)$
   (d) $(-1,1)$

4. Prove that each of the following ODEs has a unique solution through every point (x_0, y_0) in the xy-plane.
   (a) $y' = 1 + y^2$
   (b) $y' = xy^3$

5. Suppose $f(t)$ has $f''(t)$ exists for all real t. Prove $y' = f(y)$ has a unique solution through every point (x_0, y_0).