

Instructions: Legibly complete each of the following exercises; +1 bonus point if written in L^AT_EX.

1. How many integers between 1000 and 9999 inclusive are divisible by either 6 or 15?
2. A sysadmin requires accounts to have passwords of length between 10 and 13 characters. The allowed characters are lowercase and uppercase English letters, numeric digits, and special characters *, >, <, !, +, and =.
 - (a) How many passwords are possible in this scheme?
 - (b) How many passwords use at least one special character?
3. How many bit strings of length 12 contain...
 - (a) exactly 3 ones?
 - (b) at least 7 zeroes?
 - (c) the same number of zeroes and ones?
4. How many anagrams are there of the word *COMBINATORICS*?
5. How many ways can 20 people sit in order around a round table?
6. Prove that for all $1 \leq k < n$ we have $\binom{n-1}{k-1} \binom{n}{k+1} \binom{n+1}{k} = \binom{n-1}{k} \binom{n}{k-1} \binom{n+1}{k+1}$.
7. Prove that $k \binom{n}{k} = n \binom{n-1}{k-1}$ by counting the set $\{(i, S) \in [n] \times \mathbb{P}([n]) : i \in S, \#S = k\}$ in two ways.
8. Suppose 25 Apple users and 25 Windows users are seated at a round table.
 - (a) Show that someone is seated next to two Apple users.
 - (b) How long before everyone realizes they should switch to Linux?