

Math 324 Quiz 8

22 June 2016

Name: *Answer Key*

1. Compute $\mathcal{L}\{te^t\}$. $= \int_0^{\infty} te^te^{-st} dt = \int_0^{\infty} te^{(1-s)t} dt$

$$= \left[\frac{t}{1-s} e^{(1-s)t} - \frac{1}{(1-s)^2} e^{(1-s)t} \right]_0^{\infty}$$

$$= \frac{1}{(1-s)^2}$$

2. Compute $\mathcal{L}\{f(t)\}$ where $f(t) = \begin{cases} t & \text{when } 0 \leq t < 1 \\ 1 & \text{when } t \geq 1 \end{cases}$ $\left(= t + \mathcal{U}(t-1) - t\mathcal{U}(t-1) \right)$
 $= t + (1-t)\mathcal{U}(t-1)$

$$\int_0^{\infty} f(t)e^{-st} dt = \int_0^1 te^{-st} dt + \int_1^{\infty} e^{-st} dt$$

$$= \left[-\frac{t}{s} e^{-st} - \frac{1}{s^2} e^{-st} \right]_0^1 + \left[-\frac{1}{s} e^{-st} \right]_1^{\infty}$$

$$= -\frac{1}{s} e^{-s} - \frac{1}{s^2} e^{-s} + \frac{1}{s^2} + \frac{1}{s} e^{-s}$$

$$= \frac{1}{s^2} - \frac{1}{s^2} e^{-s}$$