Math 324 Quiz 5

15 June 2016

Name: Answer Key

1. Given that $y_1 = x$ is a solution to $xy'' - xy' + y = 0$, find the general solution.

$y_2 = uy_1 = ux$

$y_2' = u'x + u$

$y_2'' = u''x + 2u'$

$0 = x^2u'' + 2xu' - x^2u' - xu + xu$

$= x^2u'' + (2x - x^2)u'$. let $v = u'$.

$v' + \left(\frac{2}{x} - 1\right)v = 0$

integrating factor $= e^{\int \frac{2}{x} - 1\,dx} = e^{\ln(x^2) - x} = x^2 e^{-x}$

$(x^2e^{-x}v)' = 0$

$u' = v = x^{-2}e^{x} = \sum_{k=0}^{\infty} \frac{1}{k!} x^{k-2} = \frac{1}{x^2} + \frac{1}{x} + \frac{1}{2} + \frac{1}{3!}x + \frac{1}{4!}x^2 + \ldots$

$u = \frac{-1}{x} + \ln(x) + \frac{1}{2}x + \frac{1}{2\cdot 3!}x^2 + \frac{1}{3\cdot 4!}x^3 + \ldots$

$y = c_1y_1 + c_2y_2$

$y_2 = x\ln(x) - 1 + \frac{1}{2}x^2 + \frac{1}{2\cdot 3!}x^3 + \frac{1}{3\cdot 4!}x^4 + \ldots$

2. Find the general solution to $x^3y''' - 6y = 0$.

Cauchy-Euler

$r(r-1)(r-2) - 6 = 0$

$\shortparallel$

$r(r^2 - 3r + 2) - 6$

$\shortparallel$

$r^3 - 3r^2 + 2r - 6 = (r-3)(r^2+2)$

3 is a root, so

divide by $r-3$

$$\begin{array}{r|l} & r^2 + 2 \\ \hline r-3 & r^3 - 3r^2 + 2r - 6 \\ & -(r^3 - 3r^2) \\ & 2r - 6 \end{array}$$

so $y_1 = x^3$

$y_2 = \cos(\sqrt{2}\ln(x))$

$y_3 = \sin(\sqrt{2}\ln(x))$

$y = c_1y_1 + c_2y_2 + c_3y_3$