# Math 324 Quiz 3

6 June 2016

Name: Answer key

1. Find a solution to $y'' + 8y' + 16y = 0$ that satisfies $y(0) = 2, y'(0) = 3$.

$$r^2 + 8r + 16 = 0$$
$$\parallel$$
$$(r+4)(r+4)$$

roots $= -4, -4$

$$y = c_1 e^{-4x} + c_2 x e^{-4x}$$

$$y' = -4c_1 e^{-4x} + c_2 e^{-4x} - 4c_2 x e^{-4x}$$
$$= (c_2 - 4c_1)e^{-4x} - 4c_2 x e^{-4x}$$

$$\boxed{y = 2e^{-4x} + 11xe^{-4x}}$$

$$2 = y(0) = c_1 \longrightarrow c_1 = 2$$

$$3 = y'(0) = c_2 - 4c_1 = c_2 - 8 \longrightarrow c_2 = 11$$

2. Consider the solutions $y_1 = e^{2ix}$ and $y_2 = \cancel{\cos(2x)}$ $\sin(2x)$ to the differential equation $y'' + 4y = 0$.

(a) Compute the Wronskian $W(y_1, y_2)$.

$$W = \begin{vmatrix} e^{2ix} & \sin(2x) \\ 2ie^{2ix} & 2\cos(2x) \end{vmatrix} = 2e^{2ix}\cos(2x) - 2ie^{2ix}\sin(2x)$$
$$= 2e^{2ix}\left(\cos(2x) - i\sin(2x)\right)$$
$$= 2e^{2ix}\left(\cos(-2x) + i\sin(-2x)\right) = 2e^{2ix}e^{-2ix} = 2$$

(b) Is $y = c_1y_1 + c_2y_2$ a general solution to $y'' + 4y = 0$? Why or why not?

yes. Since the Wronskian is never 0, the two solutions are linearly independent. Therefore $y = c_1y_1 + c_2y_2$ is a general solution.