

Math 324 Quiz 2

3 June 2016

Name: Answer Key

1. Find the most general solution to $\cos(x)y' + \sin(x)y = 1$.

$$y' + \tan(x)y = \sec(x)$$

$$\int \tan(x)dx = \int \frac{\sin(x)}{\cos(x)} dx = - \int \frac{1}{u} du = -\ln(\cos(x)) = \ln(\sec(x))$$

$u = \cos(x)$
 $du = -\sin(x)dx$

$$(\sec(x)y)' = \sec^2(x)$$

$$\sec(x)y = \tan(x) + C$$

$$y = \boxed{\sin(x) + C \cos(x)}$$

2. Find the solution to $x^2y' + x(x+2)y = e^x$ that satisfies $y(1) = e$.

$$y' + \frac{x^2 + 2x}{x^2} y = \frac{e^x}{x^2}$$

$$\int 1 + \frac{2}{x} dx = x + 2\ln(x) = x + \ln(x^2)$$

$$(x^2 e^x y)' = e^{2x}$$

$$e = y(1) = \frac{e}{2} + \frac{C}{e}$$

$$x^2 e^x y = \frac{1}{2} e^{2x} + C$$

$$\frac{e}{2} = \frac{C}{e} \rightarrow C = \frac{e^2}{2}$$

$$y = \frac{e^x}{2x^2} + \frac{C}{x^2 e^x}$$

$$y = \boxed{\frac{e^x}{2x^2} + \frac{e^{2-x}}{2x^2}}$$