Math 220 Section 6 Exam 1

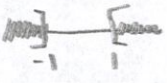
30 September 2015

Name: Answer Key

1. What is the domain of the function $f(x) = \frac{\sqrt{x^2-1}}{x+1}$? For which values of x is $f(x) = 0$?

$x+1 \neq 0$ $\quad x^2-1 \geq 0$

$x \neq -1$



the domain is $(-\infty, -1) \cup [1, \infty)$.

$f(x) = 0$

$\frac{\sqrt{x^2-1}}{x+1} = 0$

$\sqrt{x^2-1} = 0$

$x = \pm 1$ but -1 is not in domain

so $\boxed{x = 1}$

2. Compute the following limits.

(a) $\lim_{x \to 7} \frac{x^2+x+1}{x-3} = \frac{\lim_{x \to 7} x^2+x+1}{\lim_{x \to 7} x-3} = \boxed{\frac{57}{4}}$

(b) $\lim_{x \to 4} \frac{4-x}{\sqrt{x}-2} = \lim_{x \to 4} \frac{4-x}{\sqrt{x}-2} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2} = \lim_{x \to 4} \frac{(4-x)(\sqrt{x}+2)}{x-4}$

$= \lim_{x \to 4} (-1)(\sqrt{x}+2) = \boxed{-4}$

3. The cost in dollars of producing n ping-pong paddles is given by $C(n) = 2n + 180$. Each ping-pong paddle is sold for 11 dollars.

(a) What is the revenue function $R(n)$?

$R(n) = 11n$

(b) What is the profit function $P(n)$?

$P(n) = R(n) - C(n) = 11n - (2n+180) = 9n - 180$

(c) How many ping-pong paddles must be sold in order for the company to break even?

$P(n) = 0$

$9n - 180 = 0$

$n = 20$

(d) What is the marginal profit of producing 10 paddles?

$P'(n) = 9$

$P'(10) = 9$

4. Find an equation for the tangent line to the curve $f(x) = \sqrt{x^2 + 5}$ at the point $(2, 3)$.

$f'(x) = \frac{1}{2\sqrt{x^2+5}} \cdot (2x)$

$m = f'(2) = \frac{2}{\sqrt{9}} = \frac{2}{3}$

$y = \frac{2}{3}(x-2)+3$

5. Differentiate the following functions.

(a) $f(x) = \frac{xe^x}{x+1}$

$$f'(x) = \frac{(x+1)(xe^x)' - xe^x(x+1)'}{(x+1)^2} = \boxed{\frac{(x+1)(xe^x+e^x) - xe^x}{(x+1)^2}}$$

(b) $g(x) = (3x-4)^7 \ln(x^2+2x+2)$

$$g'(x) = \left((3x-4)^7\right)' \ln(x^2+2x+2) + (3x-4)^7\left(\ln(x^2+2x+2)\right)'$$

$$= \boxed{7(3x-4)^6(3)\ln(x^2+2x+2) + (3x-4)^7 \frac{2x+2}{x^2+2x+2}}$$

(c) $s(t) = e^{\sqrt{t^2+5}}$

$$s'(t) = e^{\sqrt{t^2+5}} \cdot \left(\sqrt{t^2+5}\right)'$$

$$= \boxed{e^{\sqrt{t^2+5}} \cdot \frac{1}{2\sqrt{t^2+5}} \cdot 2t}$$