

1.1 Evaluate the integral of the function $F(x, y, z) = xyz$ over the part of the unit ball lying in the first octant.

(a) using spherical coordinates

(b) using cylindrical coordinates

1.2 Given the curve $\mathbf{r}(t) = \langle 2t, t^2, \ln t \rangle$, between points $(2, 1, 0)$ and $(4, 4, \ln 2)$. Evaluate the integral $\int_C x - 2y \, ds$.

1.4 Given a vector field $\mathbf{F} = \langle 3x^2, x^3 - 2x \rangle$ and the closed curve C which is a circle of radius 2 centered at $(3, 2)$, traversed *clockwise*.

(a) Set up $\int_C \mathbf{F} \cdot d\mathbf{r}$ using the definition of the line integral and a suitable parametrization of C . **Do not calculate.**

(b) Find $\int_C \mathbf{F} \cdot d\mathbf{r}$ using Green's Theorem.

1.6 Find the integral of the function $f(x, y) = x - 3y$ over the parallelogram with vertices $(1, 2)$, $(3, 5)$, $(0, 4)$ and $(-2, 1)$.

1.7 Evaluate the integral $\iiint_D xy^2z \, dV$ where D lies above the region on the xy -plane bounded by the parabola $y = x^2$ and the line $y = 4$. And D is bounded above by the sphere of radius 5 centered at the origin.

1.8 The curve C is given by $\mathbf{r}(t) = \langle (10 + \cos(2018t)) \cos t, (10 + \cos(2018t)) \sin t \rangle$ where $0 \leq t \leq 2\pi$.

Find the integral over C of the vector field $\mathbf{F}(x, y) = \left\langle \frac{-y}{x^2 + y^2}, \frac{x}{x^2 + y^2} \right\rangle$, traversed from $t = 0$ to $t = 2\pi$.

2.2 Given the vector field $\mathbf{F} = \langle y^2, y^2 + 2xy \rangle$.

(a) Determine if it is conservative. If it is, find its potential function.

(b) Find the integral $\int_C \mathbf{F}$ over the straight segment from $A = (2, 0)$ to $B = (0, 1)$.

2.3 Find the volume of the solid defined by the inequalities

$$x^2 + y^2 + z^2 \leq 4, \quad z \geq \sqrt{x^2 + y^2}.$$

2.4 Evaluate the integral of the function $F(x, y) = x^2 + 1$ over the region $4x^2 + y^2 \leq 1$ with respect to the area.

2.8 Construct an example of a vector field $\mathbf{F} = \langle P, Q \rangle$ in some domain in the plane, such that $P_y = Q_x$ but \mathbf{F} is **not** conservative.

3.7 (a) Find the conservative vector field $\mathbf{F} = \langle P, Q, R \rangle$ such that $P = 2xz$, $Q = 2x + 3z$, $R(0, 0, 0) = 1$, R does NOT depend on z .

(b) Find a potential function for \mathbf{F} from part (a).

(c) Find the integral of \mathbf{F} from part (a) over the curve $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$, where t goes from 0 to 2.

3.8 Construct an example of a non-constant vector field \mathbf{F} on \mathbb{R}^3 such that both the divergence and the curl of \mathbf{F} are identically zero.