Permutations with Repetition

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So in general; The number of $r-$permutation of a set of n objects with repetition allowed is n^r

Combinations with repetition :

Consider the following combination problems but with repetition allowed:

Example 1 How many ways are there to select four pieces of fruit from a bowl containing apples(A), mango(M) and pears(P) if the order in which the pieces are selected does not matter, only the type of fruit matter, and there are at least four pieces of each type of fruit in the bowl?

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Examples of possible selections are: $4A$, $(3A, 1M), (2M, 1A, 1P), \ldots$ So how do we count this?

Proof.

A diagram to illustrate the problem : $\star\star|\star|\star$ this represent choosing $2A, 1M, 1P$, $\star||\star\star\star$ represent $1A, 0M, 3P$. Similarly, we can represent any other possible choices like that. So what formula count the total number of possibilities? Counting the stars and the bars, there six positions to put the stars. So number of ways to select 4 stars from 6 which is $\binom{6}{4}$

Exercises

- In how many ways can five elements be selected in order from a set with five elements when repetition is allowed?
- How many ways are there to select three unordered elements from a set with five elements when repetition is allowed.
- How many ways are there to choose eight coins from a Piggy bank containing 100 identical pennies and 80 identical nickels.

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Combinatorics with repetition (continued)

In general, there are $\binom{n+r-1}{r}$ $r-$combinations from a set with n elements when repetition is allowed.

Example: How many solutions does the equation

$$x_1 + x_2 + x_3 = 11$$

have, where x_1, x_2 and x_3 are non-negative integers?

Proof.

We can view a solution of the equation as selecting 11items from a set with 3 elements, where x_1 of item 1, x_2 of item 2 and x_3 of item 3 where chosen. So the number of ways to do this is ?

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What if there are constraints like : $x_1 \geq 1, x_2 \geq 2$ and $x_3 \geq 3$. (A solution in this case consists of $1, 2, 3$ items types $1, 2, 3$ respectively and with a choice five additional items from any of the three types: we have $\binom{(3+5-1)}{5} = \binom{7}{2}$)

more exercises

1. How many solutions are there to the equation:

$$x_1 + x_2 + x_3 + x_4 + x_5 = 21$$

   such that

   i $x_1 \geq 2$

   ii $x_i \geq 3$ for $1 \leq i \leq 5$

   iii $0 \leq x_1 \leq 10$

2. How many different bit strings can be transmitted if the string must begin with a 1 bit, must include three additional 1 bits (so that a total of four 1 bits is sent),must include a total of 14 0bits, and must have at least two 0 bits following each 1 bit.

Permutations with Indistinguishable Objects

Example: How many different strings can be made by reordering the letters of the word 'SUCCESS'

Proof.

$C(7,3) \cdot C(4,2) \cdot C(2,1) \cdot C(1,1) = \frac{7!}{3!2!}$ □