Department of Mathematical Sciences

Math 221   Calculus I   Fall 2010   Final Exam

DO NOT TURN OVER THIS PAGE UNTIL INSTRUCTED TO DO SO

NAME (Printed): _____

SECTION NO.: _____

When instructed, turn over this cover page and begin the test. You will have 2 hours to complete the test. If you have any questions, raise your hand and wait for the proctor to come to your seat.

This test contains 10 problems.

Write your work on the test paper. If you need extra space, use the blank page 12 (and indicate to the grader where to find your answer).

Unless otherwise instructed, you must show your work for each problem. Solutions with no supporting work may receive no credit. Numerical answers should be presented as exact mathematical expressions, not by a decimal approximation- for example, π rather than 3.14. CALCULATORS ARE NOT ALLOWED.

FOR GRADING PURPOSES. DO NOT WRITE IN THE SPACES BELOW.

Problem	Credit
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Total	

(1) In each of the following calculate $\frac{dy}{dx}$.

    (a) $y = (2x + 1)^{201}$.

    (b) $y = \dfrac{x^2 + 1}{x^2 - 1}$.

    (c) $y = \sin(2x + 1)$.

    (d) $y = \cos(\frac{\sqrt{x}+1}{\sin x})$.

    (e) $y = \displaystyle\int_1^x \left(\sqrt{\cot(\sin t)}\right) dt$.

(2) (a) Calculate $\lim_{x \to 5} \frac{x^2 - 2x + 15}{x^2 + x - 30}$

(b) Find the slope of the tangent line at the point $(1, 2)$ to the curve $x^2 + 2xy - y^2 + x = 2$

Hint for (b): implicit differentiation.

(3) An object is moving back and forth along the x-axis, starting at time $t = 0$. Its position after t seconds is $s(t) = t - 2 - 2\cos t$.

(a) What is the acceleration of the object at time t?

(b) What is the first time at which the velocity will be zero?

(For full credit, you should have no trigonometric functions in your answer; for example, if your answer contains $\sin(\frac{\pi}{4})$ you should know that this is $\frac{1}{\sqrt{2}}$.)

(4) A car is driving east down a straight road at 15 meters per second towards a junction. A police officer is parked 30 meters directly south of the junction and is pointing his radar gun at the car. How fast is the distance between the car and the police officer changing when the car is 40 meters from the junction?

(5) Find the intervals of increasing and decreasing, and the concavity, for the curve whose equation is $y = 2 - (x+8)^3$. Then sketch the curve.

(6) Find the horizontal and vertical asymptotes of the curve whose equation is $y = \frac{2x^2}{x^2-1}$.
(You are not required to sketch the curve.)

(7) 900 square centimeters of material is to be used to make an open-topped box which has a square base. What should be the dimensions of the box in order that its volume will be as large as possible?

(8) Calculate the following integrals:

(a) $\int_0^1 x^2(1 + 2x^3)^5 \, dx.$

(b) $\int x \sin(x^2) \, dx.$

(c) $\int \cos(x) \sin^6(x) \, dx.$

(9) (a) Find the area enclosed by the curves $y = x^2 - 1$, $y = -1$, $x = 1$ and $x = 2$.

(b) Find the average value of the function $y = \cos(x) + 2x$ on the closed interval $[0, \pi]$.

(10) (a) Find the volume of the solid obtained by rotating the region bounded by the curves $y = 1 - x^2$, and $y = 1 - x$ about the x-axis.

(b) Set up the integral for the volume of the solid obtained when that same region is rotated about the y-axis. (DO NOT evaluate this integral.)

SCRATCH PAPER