

# Introduction to “Flipped” Teaching in Freshman Calculus

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August 21, 2014

## 1 What is “flipped” teaching?

Actually, this term can mean somewhat different things, but the general idea is to answer the question: *What are you doing in class that could as well be done out of class, and what would you do with the time that this would free up?* In general, the idea is to move parts of class that are pure non-interactive content delivery to out-of-class reading or video watching, and then to spend more class time on real discussion and on students working while instructors coach.

The specific implementation that we’re using in Math 221 goes as follows:

1. For most class meetings, students are assigned approximately 10 minutes of videos to watch beforehand. The videos cover pretty much the same ground as the first few minutes a typical lecture would cover – introducing the basic concepts to be worked with in that lecture.
2. Students then typically have one or two webassign problems to complete before class, which should be very easy if they have watched and understood the video or read through the book.
3. At the beginning of class, the instructor takes questions and *briefly* recaps the videos. (“So, the big points in the video were \*\*\* and \*\*\*. Did everyone understand the calculation about \*\*\*? That’s really our starting point today, so if you need another example on that we should do it together...”) This shouldn’t be more than a few minutes.
4. The bulk of the class then follows a format: the instructor puts up a slide with one or more problems, talks briefly about the new concept this slide introduces, gives students as much help as instructor deems necessary to start working, and sets them loose to work. The students are organized into groups of about four, and they discuss and compare work within their group. The instructor circulates around the room helping groups at whatever level they need help.

After something like 5-15 minutes, depending on the problems, the instructor goes back to the front of the room for a discussion. Generally this involves working at least one complete problem at the board, while pointing out what a correct write-up looks like. Finally, the instructor gives the moral of that slide’s story: “So, to summarize, the goal of this problem/group of problems was for you to understand \*\*\*/be able to handle calculations like \*\*\*/etc. Does it make sense now?” (Depending on their response, you can discuss more or recommend they look at the webassign practice problems.)

Then you move on to another slide, illustrating the next concept.

5. Typically students in class will do all or most of the problems that would otherwise be homework. Indeed, much of the in-class exercises we prepare are taken directly from the WebAssign homeworks that we've assigned in past semesters for traditional lectures. We keep these available on webassign for students to practice, but for the most part we do not assign required homework.

The preparation of videos and slides is done by Course Coordinators and Calculus Directors. Instructors are given the Tex files for the slides and are free to tweak them, but this teaching method cuts down on class prep time for most instructors.

## 2 Our Fall 2013 experiment

We became interested in flipping through discussions with James Pitaressi, then a Mechanical Engineering professor who had found great success with it in his sophomore courses. While it sounded like a promising idea, we didn't find enough hard data to immediately convince us that it would improve student outcomes. So, in the fall of 2013, supported by a SUNY Innovative Instructional Technology Grant, we conducted our own experiment on this. We randomly assigned students in the Watson sections of Calculus 1 to either a class taught traditionally or a class taught in the flipped style. We did our best to give the two types of sections comparable instructors, in terms of experience and other factors, and then we gave all sections the same tests. Additionally, we tracked these students into Calculus 2 the following semester (in which all sections were taught in traditional lectures).

Details can be found in an upcoming paper.

As the data below details, our scepticism about flipped teaching was erased. Student and instructor feedback was largely very positive, and students in flipped sections did as well or better in every assessment. This effect held for both weaker students (defined as getting 23/33 or lower on the Screening Exam) and stronger students. We're especially enthusiastic that we got these positive results in our first try at flipping, when we were mostly inventing the wheel. We're looking forward to improving our setup more with feedback from new instructors.

## 3 Informal observations from instructors

Most of the flipped section instructors were extremely enthusiastic about what they saw in class. Student engagement ran high throughout the 90-minute classes, students of many different ability levels were demonstrating gratifying insight in class discussion, and instructors felt they were able to address the needs of individual students well. Additionally, daily attendance was consistently high and, thanks to the interactive classroom and caffeine, it was exceedingly rare for a student to doze off; recall that all sections were scheduled for 8:00 am.

Many students in the flipped sections reserved for weaker students were initially very resistant to the teaching style. Many of these students lacked the confidence or the mathematical foundation necessary to become self-sufficient problem solvers. They responded much better when the in-class problems were rearranged to give more basic computational problems at the beginning of the class. Indeed, throughout the semester instructors for these sections had

to skip the more subtle and conceptual classwork to spend more time on more concrete problems. Another great success with this cohort was to get them working in carefully selected groups, placing the weakest students with the most extrovert (regardless of ability) students.

On the other hand, students in the flipped sections reserved for strong students quickly embraced the teaching style. As they typically had confidence and a good mathematical foundation, these students quickly mastered basic computational problems and, as a consequence, more time was devoted to deeper conceptual problems.

### 3.1 Comments from a graduate instructor of a flipped section

“Student response to the flipped classroom varied widely, but it was rarely negative. Often students engaged with the material more voraciously, and the results paid off. The flipped method places the instructor in the role of facilitator, rather than lecturer. That has a number of consequences. It encourages students to think on their own, be critical, and believe in and encourage their own abilities, rather than focusing more on their instructor. It forces them to take risks which, at first uncomfortable, become more natural and from which they learn greatly. These are the skills we want to be teaching students for life, not just mathematics.

Though truly a student centered approach, practice of the flipped method also helps good teachers become great teachers. The value of seeing how students think about and solve problems in real time is irreplaceable, and it drives the way I think about and teach material now regardless of what techniques I’m using. It also gave me daily experience handling students in an extremely dynamic environment, responding to questions which were interesting, valuable, and all the more potent because they were fueled by student curiosity.

Another result was a change in my teaching reviews. They were more balanced, expressing the drawbacks as well as benefits, and were overall still positive. In effect, students felt more comfortable expressing themselves and sharing their critical thinking, just as they were instructed to do in class on a daily basis. This experience was confirmed by a couple other professors.”

## 4 Student opinions from surveys

Near the end of the semester students in the flipped sections were surveyed and we collected student comments along with numerical feedback. Overall, students tended to like the activities of the flipped teaching format. Below, we provide the distribution of student responses from the survey.

	Extreme Dislike	Dislike	Neutral	Like	Extreme Like
Videos	4%	14%	31%	38%	13%
Activities	1%	5%	17%	44%	33%
Overall	8%	14%	28%	28%	22%

## 5 Test results

Our testing system in Calculus 1 allowed us to look at different aspects of Calculus separately: We collected quantitative data through three means:

- We have the computer-based Skills Tests, covering basic computational skills,
- The common midterms are designed to be complementary to the Skills Tests, covering more sophisticated problem-solving,
- the common final exam covers all types of problems.

A frequent complaint about initiatives to change the teaching of Calculus is that students come out with greater conceptual understanding but inadequate computational skills, or vice-versa, or that the areas in which they gained were not what they needed to succeed in follow-up courses. By looking at the different types of test separately, and by looking at performance in Calculus 2 in the following semester, we tried to address this.

To summarize what follows: students in flipped sections did at least as well as students in non-flipped sections in each assessment. In some cases the advantage of flipped over non-flipped was not statistically significant, but, most notably, students in flipped sections did strikingly better on the final exam. While results in Calculus 2 performance are tentatively promising, the number of students involved was too small to claim statistical significance.

## 5.1 Skills Tests

The Skills Tests are intended to set a firm lower bar on passing the course. Students are given a thorough selection of practice problems in advance, they have multiple tries on each test, and the problems selected for these tests are very basic. In addition, students are highly motivated to pass these tests – they must get a score of at least 70% on each Skills Test in order to pass the class. Thus most students do pass – in fact, they typically eventually get a good grade. This limits the effectiveness of Skills Tests grades as a comparison of flipped vs. traditional format sections:

Flipped Sections					Controlled Sections				
	ST 1	ST 2	ST 3	ST 4		ST 1	ST 2	ST 3	ST 4
<b>A</b>	72%	59%	47%	66%	<b>A</b>	71%	55%	51%	69%
<b>B</b>	23%	23%	19%	17%	<b>B</b>	19%	25%	15%	16%
<b>C</b>	6%	16%	33%	16%	<b>C</b>	10%	17%	33%	14%
<b>F</b>	0%	2.3%	1.6%	0.8%	<b>F</b>	1%	3%	2%	1%

At the very least, we can say that our flipped teaching method did not impair students' basic computational skills.

## 5.2 Midterm and Final Examinations

The midterm exams were written to be complementary to the Skills Tests. They avoided basic computational questions and focused on more sophisticated problem-solving and conceptual problems. The  $P$ -values in this section were obtained using a one-tailed unpaired  $t$ -test.

The flipped and controlled sections took common Midterm Exams. With a  $P$ -value of 0.078 and 0.389, the difference between the flipped and controlled Midterm Exam mean is **not** statistically significant.

	Midterm 1 Flipped	Midterm 1 Controlled	Midterm 2 Flipped	Midterm 2 Controlled
Average	67.7	64.7	63.7	63
3 Quartile	83	79	78	78
Median	73	66	68	66
1 Quartile	56	53	52	50
SDeviation	18.9	18.2	19.2	20.1
Number	135	197	127	189

The final exam is administered simultaneously coursewide and is where computation and concepts are reunited. Students in the flipped sections did strikingly well in comparison to students in the controlled sections.

### Final Exam

	Flipped (All)	Controlled (All)	Flipped (Strong)	Controlled (Strong)	Flipped (Weak)	Controlled (Weak)
Average	69.1	61	71.8	62.2	60.3	54.7
3 Quartile	79.5	74.88	80.63	75.5	71.63	63
Median	70.75	62.5	73	64	60.5	55.5
1 Quartile	60.25	49.13	64	50.5	51.88	40.5
SDeviation	14.6	18.1	13.1	18.6	16	13.6
Number	126	186	96	156	30	29

Statistical details:

1. With a P-value of 0.002, the difference between the flipped sections and controlled sections is very statistically significant.
2. With a P-value of 0.0007, the mean final exam difference between the strong flipped sections and strong controlled sections is very statistically significant.
3. In terms of raw scores the weaker flipped students outperformed the weaker controlled students. Because of the small number of students in the weak cohort, statistical analysis is less conclusive. With a P-value of 0.072, the difference is **not** statistically significant.

Our best explanation for why such a strong difference emerged at the final is that student engagement remained high in the flipped sections to the very end of the semester. These students did not seem to suffer the kind of burnout so frequently seen in traditional lecture courses in the later half of the semester.

### 5.3 Success in Calculus 2

The data below is for students who were part of the experiment (flipped or controlled) in the fall and went on to take Calculus 2 in the spring. All sections of Calculus 2 were taught in a traditional format. We looked only at final exam grades because this was the only assessment common to all sections of Calculus 2.

In raw numbers, students from the flipped Calculus 1 sections did better in Calculus 2. However, for every pairing, the differences in the means between flipped, controlled, and coursewide sections is not statistically significant.

## Common Final Exam - Calculus 2

	Flipped	Controlled
Average	63.4	60
3 Quartile	71.1	70
Median	64.4	60
1 Quartile	53.3	50.4
SDeviation	15.6	18.8
Number	75	63

### 6 To grad student instructors: why you should be excited about flipping

1. It works. Students are happier, instructors are happier, everyone stays awake all semester, grades are higher.
2. Running these classes is fun. You get much more engagement with your students. Instead of lecturing to an imaginary median student, you're chatting with your whiz kids about what the multivariable version of all this will look like, helping your average students discover computational subtleties, and helping your struggling students fill their Pre-Calculus gaps.
3. Your class prep time is lower and more productive. Instead of chugging out lecture notes on boilerplate stuff, you're thinking about the nuances of the mathematics and how they'll come up in discussion.
4. It does wonders for your own teaching. To repeat an instructor quote from earlier, "The value of seeing how students think about and solve problems in real time is irreplaceable, and it drives the way I think about and teach material now regardless of what techniques I'm using."
5. Flipped teaching is *very* fashionable these days. Having this kind of experience on your teaching statement in job applications will make hiring committees sit up and take notice.