Verify that the function satisfies the Mean Value Theorem on the given interval. Then find all numbers c which satisfy the conclusion of the Mean Value Theorem.

a) $f(x) = 3x^2 + 2x + 5$ on $[-1, 1]$.

b) $g(x) = x^3 + x - 1$ on $[0, 2]$.

c) $h(x) = \dfrac{x}{x+2}$ on $[1, 4]$.

d) $i(x) = (x-2)^{-2}$ on $[1, 4]$.

On a toll road a driver takes a time stamped toll-card from the starting booth and drives directly to the end of the toll section. After paying the required toll, the driver is surprised to receive a speeding ticket along with the toll receipt. Which of the following describes the situation?

a) The booth attendant does not have enough information to prove that the driver was speeding.

b) The booth attendant can prove that the driver was speeding during their trip.

c) The driver will get a ticker for a lower speed than their actual maximum speed.

True or False

An athlete is running back and forth along a straight path. She finishes her run at the place where she began. There must be at least one moment, other than the end of the race, where she was at a complete stop.

Two runners start a race at the same moment and finish in a tie. What must be true?

a) At some point during the race the two runners were not tied.

b) The runners' speeds at the end of the race must have been exactly the same.

c) The runners must have had the same speed at exactly the same time at some point in the race.

d) The runners had to have the same speed at some moment, but not necessarily at exactly the same time.

Show that for all values a and b

$$|\sin(a) - \sin(b)| \leq |a - b|$$

Suppose that $3 \leq f'(x) \leq 5$ for all values of x. Show that

$$18 \leq f(8) - f(2) \leq 30$$

Show that the polynomial

$$f(x) = 1 + 2x + x^3 + 4x^5$$

has exactly one real root.