Important Information about Sample Final Exams

In the following pages are 5 sample final exams with answers. There are problems on these sample finals which we did not cover in class, and in turn do not need to do. Please ignore the following problems:

Sample 1: do not do #1, #2, #6
Sample 2: do not do #2(a), #6
Sample 3: do not do #1, #2, #5
Sample 4: do not do #4
Sample 5: do not do #5(b), #8

Calculus 3 Final Sample Exam 1

courtesy of Dr. Inna Sysoeva, University of Pittsburgh, adapted

Problem 1. Find the unit tangent and unit normal vectors $\mathbf{T}$ and $\mathbf{N}$ to the curve $\mathbf{r}(t) = \langle 3\cos t,\, 4t,\, 3\sin t\rangle$ at the point $P = \left(-\frac{3}{\sqrt{2}},\, 3\pi,\, \frac{3}{\sqrt{2}}\right)$.

(b) Find curvature of the curve at the point P.

Problem 2. Use LINEAR approximation to approximate the number $\sqrt{3.04 + e^{-0.08}}$.

Problem 3. Find all critical points of the function $f(x, y) = 4x - 3x^3 - 2xy^2$. For each critical point determine if it is a local maximum, local minimum or a saddle point.

Problem 4. Find the volume of the solid E bounded by $y = x^2$, $x = y^2$, $z = x+y+5$, and $z = 0$.

Problem 5. Find the integral of the function $f(x, y) = y^2$ on the region bounded by $y^2 = x+4$, $x = 0$, and $y \geq 0$. Simplify your answer as much as possible.

Problem 6. Evaluate the integral $\iint\limits_R e^{x-2y}\, dA$ where R is the parallelogram $ABCD$ with vertices $A = (0, 0)$, $B = (4, 1)$, $C = (7, 4)$, and $D = (3, 3)$ using the transformation $x = 4u + 3v$ and $y = u + 3v$. Simplify your answer as much as possible.

Problem 7. Evaluate the line integral $\oint\limits_C e^{2x+y}\, dx + e^{-y}\, dy$ along the **negatively** oriented closed curve C, where C is the boundary of the triangle with the vertices $(0, 0)$, $(0, 1)$, and $(1, 0)$.

Problem 8. Evaluate the integral $\iint\limits_S (10 - 2z)\, dS$, where S is the part of the surface $z = 5 - \frac{x^2}{2} - \frac{y^2}{2}$ inside the cylinder $x^2 + y^2 = 1$.

Problem 9. Evaluate the line integral $\oint\limits_C \mathbf{F} \cdot d\mathbf{r}$ for the vector field $\mathbf{F}(x, y, z) = -y\,\mathbf{i} + x\,\mathbf{j} - z\,\mathbf{k}$, where the closed curve C is the boundary of the triangle with vertices $(0, 0, 5)$, $(2, 0, 1)$, and $(0, 3, 2)$ traced in this order.

Problem 10. Evaluate the flux of $\mathbf{F}(x, y, z) = z^2 y\,\mathbf{i} + x^2 y\,\mathbf{j} + (x + y)\,\mathbf{k}$ over S, where S is the closed surface consisting of the coordinate planes and the part of the sphere $x^2 + y^2 + z^2 = 4$ in the first octant $x \geq 0$, $y \geq 0$, $z \geq 0$, with the normal pointing outward.

Calculus 3 Final Examination Sample 1 - ANSWERS

courtesy of Dr. Inna Sysoeva, University of Pittsburgh, adapted

Problem 1. a) $\mathbf{T} =< -\frac{3}{5\sqrt{2}}, \frac{4}{5}, -\frac{3}{5\sqrt{2}} >$, $\mathbf{N} =< \frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}} >$
b) $\frac{3}{25}$

Problem 2. 1.99.

Problem 3. $(0, \sqrt{2})$ and $(0, -\sqrt{2})$ - saddles;
$(\frac{2}{3}, 0)$ - local maximum
$(-\frac{2}{3}, 0)$ - local minimum

Problem 4. $\frac{59}{30}$

Problem 5. $\frac{64}{15}$

Problem 6. $\frac{3}{2}(e^2 - 1 - \frac{1}{e} + \frac{1}{e^3})$

Problem 7. $\frac{e^2}{2} - e + \frac{1}{2}$

Problem 8. $\frac{4\pi(\sqrt{2}+1)}{15}$

Problem 9. 6.

Problem 10. $\frac{16\pi}{15}$

Calclulus 3 Final Sample Exam 2

courtesy of Dr. Inna Sysoeva, University of Pittsburgh, adapted

Problem 1. Function f is given by the formula $f(x, y) = 2x^2 + 3e^{xy}$.

a) Find the directional derivative of f at the point $P = (1, 0)$ in the direction of the vector $\mathbf{u} =< -1, 2 >$.

b) Find the maximal rate of change of $f(x, y)$ at P and the direction in which it occurs.

Problem 2. The curve is given parametrically by $\mathbf{r}(t) =< t^3 + \frac{1}{2}t^2, 2t - 1, t^2 + t\sqrt{5} >$.

a) Find its curvature at the point $(0, -1, 0)$.

b) Set up the integral representing the length of the curve from the point $(0, -1, 0)$ to the point $(10, 3, 4 + 2\sqrt{5})$.

DO NOT EVALUATE THE INTEGRAL.

Problem 3. Find an equation of the plane tangent to the surface $x^2 + y^2z^2 = 8$ at the point $P = (2, 2, 1)$.

Problem 4. Find all critical points of the function $f(x, y) = x^2 + 4xy - 10x + y^2 - 8y + 1$. For each critical point determine if it is a local maximum, a local minimum or a saddle point.

Problem 5. Find the work done by the force $\mathbf{F}(x, y) = 3y\mathbf{i} + x\mathbf{j}$ in moving a particle along the boundary of the trapeziod with the vertices $(0, 0)$, $(1, 1)$, $(2, 1)$ and $(3, 0)$ in the clockwise direction.

Problem 6. Find the mass of the solid bounded by the surfaces $y^2 + z^2 = 1$, $x = 0$ and $x = y^2 + z^2 - 4$, if the density function is given by the formula $\rho(x, y, z) = y^2 + z^2$.

Problem 7. a) Determine whether the vector field $\mathbf{F}(x, y, z) = (2y + 4z)\mathbf{i} + (2x + 3z)\mathbf{j} + (4x + 3y)\mathbf{k}$, is conservative or not.

b) Evaluate $\int\limits_C (2y + 4z)dx + (2x + 3z)dy + (4x + 3y)dz$, where C is the curve given by $\mathbf{r}(t) =< t^3, 2\sin\left(\frac{\pi t}{2}\right), 3\cos\left(\frac{\pi t}{2}\right) >$ for $0 \le t \le 1$.

Problem 8. Find the maximum and minimum values of the function $F(x, y, z) = x - y$ on the $x^2 + y^2 + xy + z^2 = 1$

Problem 9. Evaluate $\oint\limits_C \mathbf{F} \cdot d\mathbf{r}$, if $\mathbf{F}(x, y, z) = y\mathbf{i} + 2x\mathbf{j} + yz\mathbf{k}$, and C is the curve of intersection of the part of the paraboliod $z = 1 - x^2 - y^2$ in the first octant $(x \ge 0,\ y \ge 0,\ z \ge 0)$ with the coordinate planes $x = 0$, $y = 0$ and $z = 0$, oriented counterclockwise when viewed from above.

Problem 10. Evaluate $\iint\limits_S \mathbf{F} \cdot d\mathbf{S}$, if $\mathbf{F}(x, y, z) = (yz)\mathbf{i} + (x^2y)\mathbf{j} + (4zx^2)\mathbf{k}$ and S is the surface of the solid bounded by the upper hemisphere $x^2 + y^2 + z^2 = 1$, $z \ge 0$, and the plane $z = 0$ with the normal pointing outward.

Calculus 3 Final Examination Sample 2 - ANSWERS

courtesy of Dr. Inna Sysoeva, University of Pittsburgh, adapted

Problem 1. a) $\frac{2}{\sqrt{5}}$
b)$< \frac{4}{5}, \frac{3}{5} >$ (or $< 4, 3 >$)

Problem 2. a) $\frac{5}{27}$
b) $\int_0^2 \sqrt{(3t^2+t)^2+4+(2t+\sqrt{5})^2}\, dt$

Problem 3. $(x-2)+(y-2)+2(z-1)=0$

Problem 4. $(1,2)$ - saddle point

Problem 5. 4

Problem 6. $\frac{5\pi}{3}$

Problem 7. a) conservative;
b) 4

Problem 8. Absolute minimum is -2 at $(-1,1,0)$;
Absolute maximum is 2 at $(1,-1,0)$

Problem 9. $\frac{\pi}{4}+\frac{4}{15}$

Problem 10. $\frac{2\pi}{3}$

Calculus 3 Final Sample Exam 3

courtesy of Dr. Inna Sysoeva, University of Pittsburgh, adapted

Problem 1. a) Find the unit tangent vector $\mathbf{T}$ and unit normal vector $\mathbf{N}$ to the curve $\mathbf{r}(t) =< 3\cos t, 4t, 3\sin t >$ at the point $P = \left(-\frac{3}{\sqrt{2}}, 3\pi, \frac{3}{\sqrt{2}}\right)$.

b) Find the curvature of the curve at the point P.

Problem 2. Use linear approximation to approximate the number $\sqrt{3.04 + e^{-0.08}}$.

Problem 3. Determine all local maxima, local minima and saddle points of $f(x, y) = 3y - y^3 - 3x^2y$.

Problem 4. A rectangular box without a lid is to be made from 48 ft^2 of cardboard. Find the maximum volume of the box.

Problem 5. Find the $y-$ coordinate of the center of mass of a lamina that occupies the region bounded by $y^2 = x + 4,\ x = 0$, and $y \geq 0$ and has density $\rho(x, y) = y$. Simplify your answer as much as possible.

Note: The $y-$ coordinate of the center of mass of a lamina with density function ρ is $(\iint\limits_D y\rho\, dA)/(\iint\limits_D \rho\, dA)$.

Problem 6. Find the volume of the solid that lies within the cylinder $x^2 + y^2 = 4$, above the $(x, y)-$plane, and below the cone $z^2 = 4x^2 + 4y^2$.

Problem 7. Let $\mathbf{F}$ be the two-dimensional vector field given by $\mathbf{F}(x, y) =< ye^{xy} - 1, xe^{xy} + 2y >$.

a) Determine if $\mathbf{F}$ is a conservative vector field, and if so, find a potential function.

b) Find the value of the line integral $\int\limits_C \mathbf{F}\cdot\mathbf{T}ds$, where C is the line segment from $(0, 3)$ to $(5, 0)$.

Problem 8. Use Green's Theorem to find the value of $\oint\limits_C -5x^2\, dx + 7xy\, dy$, where C is the closed curve consisting of the edges of the triangle with vertices $(0, 0),\ (3, 1)$, and $(0, 3)$, oriented counterclockwise.

Problem 9. Use Divergence Theorem to find the total flux $\iint\limits_S \mathbf{F}\cdot\mathbf{n}\, dS$ of the vector field $\mathbf{F}(x, y, z) =< x^2, yz^2, -2xz >$ across the surface S given by $x^2 + y^2 + z^2 = 2$ with outward orientation.

Problem 10. Use Stokes' theorem to evaluate $\oint\limits_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = e^x\mathbf{i} + (x^2 + y^2)\mathbf{j} + z\mathbf{k}$, and C is the boundary of the part of the plane $2x + y + 2z = 2$ in the first octant oriented counterclockwise when viewed from above.

Calculus 3 Final Examination Sample 3 - ANSWERS

courtesy of Dr. Inna Sysoeva, University of Pittsburgh, adapted

Problem 1. a) $\mathbf{T} =< -\frac{3\sqrt{2}}{10}, \frac{4}{5}, -\frac{3\sqrt{2}}{10} >$, $\mathbf{N} =< \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} >$
b) $\frac{3}{25}$

Problem 2. 1.99.

Problem 3. $(1,0)$ and $(-1,0)$ - saddles;
$(0,1)$ - local maximum
$(0,-1)$ - local minimum

Problem 4. $32\ ft^2$

Problem 5. $\frac{16}{15}$

Problem 6. $\frac{32\pi}{3}$

Problem 7. a) conservative; $f(x,y) = e^{xy} + y^2 - x$
b) -14

Problem 8. 42

Problem 9. $\frac{16\pi\sqrt{2}}{15}$

Problem 10. $\frac{2}{3}$

Calculus 3 Final Sample Exam 4

courtesy of Dr. Inna Sysoeva, University of Pittsburgh, adapted

Problem 1. Show that the points $A\,(2,1,3), B\,(0,5,5), C\,(3,6,9)$ and $D\,(5,2,7)$ are the vertices of a parallelogram P and calculate the area of P.

Problem 2. Determine whether the lines
$L_1 : x = t + 2, y = 3t + 1, z = t + 3$, $L_2 : x = -2s + 3, y = -2, z = 4s - 2$, intersect. If they intersect, find the point(s) of intersection. If they do not intersect, are they parallel?

Problem 3. Let S be the level surface defined by $x^2 - y^2 + z^2 = 1$.

(a) Find the equations of the tangent plane and normal line to S at the point $(1, 1, -1)$.

(b) Find all the points of intersection of the line found in (a) and the surface S (of course, $(1, 1, -1)$ is one of them).

Problem 4. The position of a particle at time t is given by the function $\mathbf{r}(t) = \left\langle \frac{1}{3}t^3, 2t\sin t, 2t\cos t \right\rangle, t \geq 0$. Find the speed $v(t)$ of the particle, the tangential component $a_T(t)$ of its acceleration and the distance traveled (arc length) between the times $t = 0$ and $t = 3$.

Problem 5. Let $z = xy, x = uv, y = vw$. Use the Chain Rule to find $\frac{\partial z}{\partial u}, \frac{\partial z}{\partial v}$ and $\frac{\partial z}{\partial w}$. Give your answers in terms of the variables u, v and w alone.

Problem 6. Let $f(x, y) = (x - 3)^2 + (y + 3)^2$.

(a) Find the minimum and maximum values of f under the constraint $x^2 + y^2 = 8$.

(b) Find the minimum and maximum values of f in the disk with center $(0, 0)$ and radius $\sqrt{8}$.

Problem 7. Evaluate $\int\int_D 4y^2 dA$ where D is the intersection of the unit disk with the first quadrant: $D = \{(x, y) | x \geq 0, y \geq 0, x^2 + y^2 \leq 1\}$.

Problem 8. Determine whether the field $\mathbf{F}$ is conservative. If it is, evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the plane curve with equation $\mathbf{r}(t) = \langle \cos t, \sin t \rangle$, $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$.

(a) $\mathbf{F}(x, y) = x^2 \cos y\,\mathbf{i} + 2x \sin y\,\mathbf{j}$.

(b) $\mathbf{F}(x, y) = (\sin x + \cos y)\,\mathbf{i} + (2 - x\sin y)\,\mathbf{j}$.

Problem 9. Let D be a plane region whose boundary C is a simple smooth closed curve. Assume that the area of D is 3 and that C is given the clockwise orientation. Evaluate $\int_C (xy^2 + 2y)dx + x^2 y dy$. (You are not allowed to make any specific choice for D.)

Problem 10. Find the flux $\int\int_S \mathbf{F} \cdot d\mathbf{S}$ of the constant vector field $\mathbf{F}(x, y, z) = \langle 0, 0, 1 \rangle$ across the surface S with vector equation
$\mathbf{r}(u, v) = \langle u^2 - v, u + v^2, uv \rangle, 0 \leq u \leq 1, 0 \leq v \leq 1$.

Calculus 3 Final Examination Sample 4 - ANSWERS

courtesy of Dr. Inna Sysoeva, University of Pittsburgh, adapted

Problem 1. Check that the vectors forming opposite sides of $ABCD$ are equal. The area is $14\sqrt{3}$.

Problem 2. The lines intersect at $(1, -2, 2)$.

Problem 3. a) Plane: $x - y - z = 1$;
line: $x = 1 + t,\ y = 1 - t,\ z = -1 - t$
b) $(1, 1, -1)$ and $(-1, 3, -3)$.

Problem 4. $v(t) = t^2 + 2$
$a_T = 2t$
$d = 15$

Problem 5. $\frac{\partial z}{\partial u} = v^2 w, \quad \frac{\partial z}{\partial v} = 2uvw, \quad \frac{\partial z}{\partial w} = uv^2$

Problem 6. Answer for both parts a) and b):
Absolute maximum is 50 at $(-2, 2)$;
Absolute minimum is 2 at $(2, -2)$

Problem 7. $\frac{\pi}{8} - \frac{1}{4}$

Problem 8. a) Not conservative;
b) Conservative; 4

Problem 9. 6

Problem 10. 2

Calculus 3 Final Sample Exam 5

courtesy of Dr. Inna Sysoeva, University of Pittsburgh, adapted

Problem 1. Find the points on the cone $z^2 = x^2+y^2$ closest to the point $(1, \sqrt{3}, 4)$.

Problem 2. Express the volume of the tetrahedron with vertices at points $(0, 0, 0)$, $(1, 0, 0)$, $(0, 2, 0)$, $(0, 0, 3)$ as a triple integral. Write out the limits of integration explicitly, but do not evaluate.

Problem 3. Write the equation of any plane tangent to the cone $z^2 = x^2 + y^2$.

Problem 4. Use Stokes' theorem to evaluate $\int\limits_C \mathbf{F} \cdot d\mathbf{r}$, where $F(x, y, z) = e^x\mathbf{i} + (x^2 + y)\mathbf{j} + z\mathbf{k}$, and C is the boundary of the part of the plane $2x + y + 2z = 2$ in the first quadrant oriented counterclockwise as viewed from above.

Problem 5. Consider the helix $r(t) =< \cos t, \sin t, t >,\ 0 \le t \le \pi$.

a) Find the length of this helix.

b) If the helix has density z at point (x, y, z), what is its mass?

Problem 6. Let D be the region $D = \{(x, y, z) : x^2 + y^2 + z^2 \le 2,\ z \ge x^2 + y^2\}$ and S be the surface of D with outward pointing normal.

a) Calculate the volume of D.

b) Let $\mathbf{F}(x, y, z) =< yz^2, 3x+z \cos x, x^2y^3 >$. Use the divergence theorem and the result in part a) to find $\iint\limits_S \mathbf{F} \cdot d\mathbf{S}$.

Problem 7. Let S be the lateral surface of the cone $z^2 = x^2+y^2$, between $z = 0$ and $z = 1$ with outward orientation, and let $\mathbf{F}(x, y, z) =< -y, x, 0 >$. Denote by I the value of the integral $\iint\limits_S curl(\mathbf{F}) \cdot d\mathbf{S}$.

a) Compute $curl(\mathbf{F})$ and $div(\mathbf{F})$.

b) Find I by direct evaluation.

c) Find I by using Stokes' theorem.

d) Find I by use of the divergence theorem. (Hint: Cover the top of the cup by a flat circular disk.)

Problem 8. Find two non-zero vectors $\mathbf{v}$ and $\mathbf{w}$, such that $\mathbf{v} \cdot \mathbf{w} = 0$ and both $\mathbf{v}$ and $\mathbf{w}$ are perpendicular to the line $x = y = z$.

Problem 9. Find the limit, if it exists:

$$\lim_{(x,y)\to(0,0)} \frac{2009x^2 - y^2}{1 - \sqrt{1 + 2009x^2 - y^2}}$$

Problem 10. Let $\mathbf{F}(x, y, z) =< xy - 3, -y^2, yz - z >$. Evaluate in the simplest way possible the surface integral $\int\limits_S\int \mathbf{F} \cdot \mathbf{n} dS$, where S is the boundary of the unit cube $\{(x, y, z) : 0 \le x, y, z \le 1\}$, and $\mathbf{n}$ denotes the inward unit normal to S.

Calculus 3 Final Examination Sample 5 - ANSWERS

courtesy of Dr. Inna Sysoeva, University of Pittsburgh, adapted

Problem 1. $(\frac{3}{2}, \frac{3\sqrt{3}}{2}, 3)$

Problem 2. There are 6 possible correct answers. One of the possible answers is:

$$\int_0^1 \int_0^{(2-2x)} \int_0^{(1-\frac{y}{2}-\frac{z}{3})} dz\ dy\ dx$$

Problem 3. There are infinitely many possible answers. An example of the correct answer: $x - z = 0$.

Problem 4. $\frac{2}{3}$

Problem 5. a) $\sqrt{2}\pi$; b) $\frac{\pi^2}{\sqrt{2}}$

Problem 6. a) $\frac{4\pi\sqrt{2}}{3} - \frac{7\pi}{6}$; b) 0.

Problem 7. a) $< 0, 0, 2 >$ and 0; The answer for b),c), d): $\frac{4\pi}{3}$

Problem 8. There are many possible correct answers here. A possible choice: $\mathbf{v} =< 1, -1, 0 >$ and $\mathbf{w} =< 1, 1, -2 >$

Problem 9. -2

Problem 10. 0