Problem 1. (10 points) Find the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{3^n}{n}(x-2)^n$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{3^{n+1}|x-2|^{n+1}}{n+1} \cdot \frac{n}{3^n|x-2|^n} = \lim_{n\to\infty} \frac{3^{n+1}}{3^n} \cdot \frac{|x-2|^{n+1}}{|x-2|^n} \cdot \frac{n}{n+1}$$

$$= \lim_{n\to\infty} 3|x-2|\left(\frac{n}{n+1}\right)$$

$$= 3|x-2| < 1$$

$$= |x-2| < \frac{1}{3} \quad \boxed{R = \frac{1}{3}}$$

$$-\frac{1}{3} < x-2 < \frac{1}{3}$$

$$\frac{5}{3} < x < \frac{7}{3} \quad \boxed{I = [5/3, 7/3)}$$

$x = 5/3$: $\sum_{n=1}^{\infty} \frac{3^n}{n}(-1/3)^n$

$= \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ Alt. Harmonic "Convergent"

$x = 7/3$: $\sum_{n=1}^{\infty} \frac{3^n}{n}(1/3)^n = \sum_{n=1}^{\infty} \frac{1}{n}$

Harmonic "divergent"

Problem 2. (10 points) Use the power series method to find the limit .

$$\lim_{x\to 0} \frac{xe^{x^2} - \sin x}{\arctan(2x^3)} = \lim_{x\to 0} \frac{x\sum_{n=0}^{\infty}\frac{x^{2n}}{n!} - \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)!}}{\sum_{n=0}^{\infty}\frac{(-1)^n(2x^3)^{2n+1}}{2n+1}}$$

$$= \lim_{x\to 0} \frac{\sum \frac{x^{2n+1}}{n!} + \sum \frac{(-1)^{n+1}x^{2n+1}}{(2n+1)!}}{\sum \frac{(-1)^n 2^{2n+1} x^{6n+3}}{2n+1}}$$

$$= \lim_{x\to 0} \frac{(\cancel{x} + x^3 + \frac{x^5}{2} + \dots) + (\cancel{-x} + \frac{x^3}{3!} - \frac{x^5}{5!} + \dots)}{2x^3 - \frac{8x^9}{3} + \dots} = \lim_{x\to 0} \frac{(1+\frac{1}{6})x^3 + (\frac{1}{2} - \frac{1}{5!})x^5 + \dots}{2x^3 - \frac{8}{3}x^9 + \dots}$$

$$= \lim_{x\to 0} \frac{(1+1/6) + (\frac{1}{2} - \frac{1}{5!})x^2 + \dots}{2 - 8/3\, x^6 + \dots} = \frac{7/6}{2} = \boxed{7/12}$$

Problem 3. (5 points) Suppose $\sum_{n=0}^{\infty} a_n(x-4)^n$ converges conditionally at $x = 15$. What is the radius of convergence of the series $\sum_{n=0}^{\infty} 2^n a_n(x-3)^n$?

$\sum_{n=0}^{\infty} a_n(15-4)^n = \sum_{n=0}^{\infty} a_n(11)^n$ is <u>Conditionally</u> conv. so using ratio test we have $\lim_{n\to\infty}\left|\frac{a_{n+1}(11)^{n+1}}{a_n(11)^n}\right| = 11\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1 \Rightarrow \lim\left|\frac{a_{n+1}}{a_n}\right| = 1/11.$

$$\lim_{n\to\infty} \frac{2^{n+1}|a_{n+1}||x-3|^{n+1}}{2^n|a_n||x-3|^n} = \lim_{n\to\infty} 2\left|\frac{a_{n+1}}{a_n}\right||x-3| = 2|x-3|\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$$

$$= 2|x-3|\left(\tfrac{1}{11}\right) < 1$$

$$|x-3| < \frac{11}{2}$$

$$\boxed{R = \frac{11}{2}}$$

Problem 4. (20 points) The Binomial Series formula can be used to show that

$$(1+x)^{-1/2} = \sum_{n=0}^{\infty} \frac{(-1)^n (2n)!}{4^n (n!)^2} x^n$$

a) Find the radius of convergence of this series.

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{(2n+2)!\,|x|^{n+1}}{4^{n+1}[(n+1)!]^2} \cdot \frac{4^n (n!)^2}{(2n)!\,|x|^n} = \lim_{n\to\infty} \frac{(2n+2)!}{(2n)!} \cdot \frac{|x|^{n+1}}{|x|^n} \cdot \frac{4^n}{4^{n+1}} \left(\frac{n!}{(n+1)!}\right)^2$$

$$= |x| \lim_{n\to\infty} \frac{(2n+1)(2n+2)}{4} \left(\frac{1}{n+1}\right)^2$$

$$\boxed{R=1}$$

$$= |x| \lim_{n\to\infty} \frac{4n^2+6n+2}{4n^2+8n+4} = |x| < 1$$

b) Use this formula to obtain the Maclaurin series for $\frac{1}{\sqrt{1-x^2}}$

$$\frac{1}{\sqrt{1-x^2}} = \sum_{n=0}^{\infty} \frac{(-1)^n (2n)!}{4^n (n!)^2} (-x^2)^n = \sum_{n=0}^{\infty} \frac{(-1)^n (-1)^n (2n)!\, x^{2n}}{4^n (n!)^2}$$

$$= \sum_{n=0}^{\infty} \frac{(2n)!}{4^n (n!)^2} x^{2n}$$

c) Use the result of part b) to obtain the Maclaurin series for $\arcsin x$.

$$\arcsin x = \int \frac{1}{\sqrt{1-x^2}}\,dx = \int \left[\sum_{n=0}^{\infty} \frac{(2n)!}{4^n (n!)^2} x^{2n}\right] dx$$

$\arcsin(0) = 0 + C$

So, $C = 0$

$$= \sum \int \frac{(2n)!}{4^n (n!)^2} x^{2n}\,dx$$

$$= \sum_{n=0}^{\infty} \frac{(2n)!\, x^{2n+1}}{4^n (2n+1)(n!)^2} + C$$

d) Use the result of part c) to find the sum of the series

$$\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2 \cdot 16^n \cdot (2n+1)} = \boxed{\frac{\pi}{3}}$$

$$\arcsin(1/2) = \sum_{n=0}^{\infty} \frac{(2n)!\,(1/2)^{2n+1}}{4^n (2n+1)(n!)^2} = \frac{1}{2} \sum_{n=0}^{\infty} \frac{(2n)!\,(1/4^n)}{4^n (2n+1)(n!)^2}$$

$$= \frac{1}{2} \sum_{n=0}^{\infty} \frac{(2n)!}{(16)^n (2n+1)(n!)^2}$$

$$2 \arcsin(1/2) = \sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2\, 16^n (2n+1)}$$

$$2\left(\frac{\pi}{6}\right) = \sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2\, 16^n (2n+1)}$$

Problem 5. (10 points)

a) Find the Maclaurin series for $f(x) = \cos(2x^3)$

$$\cos(2x^3) = \sum_{n=0}^{\infty} \frac{(-1)^n (2x^3)^{2n}}{(2n)!}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n}}{(2n)!} x^{6n}$$

b) Find $f^{(12)}(0)$

$$\sum_{n=0}^{\infty} C_n x^n \text{ where} \rightarrow C_n = \frac{f^{(n)}(0)}{n!} \Rightarrow f^{(n)}(0) = C_n \cdot n!$$

$$f^{(12)}(0) = C_{12} \cdot (12)!$$

$$= \frac{2^4}{4!} \cdot 12!$$

$$= \frac{2}{3} \cdot 12!$$

Problem 6. (10 points) Find the sum of the series.

a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \cdot 3^n}{4^n \cdot n!} = \sum_{n=0}^{\infty} \frac{(-1)^n (3/4)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-3/4)^n}{n!} = \boxed{e^{-3/4}}$

b) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n \cdot 3^n}{4^n \cdot (2n)!}$

$$= \sum_{n=2}^{\infty} \frac{(-1)^n (\sqrt{3/4})^{2n}}{(2n)!}$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n (\sqrt{3/4})^{2n}}{(2n)!} = \cos \frac{\sqrt{3}}{2}$$

$$1 - \frac{3/4}{2} + \sum_{n=2}^{\infty} \frac{(-1)^n (\sqrt{3}/2)^{2n}}{(2n)!} = \cos \frac{\sqrt{3}}{2}$$

$$\sum_{n=2}^{\infty} \frac{(-1)^n (\sqrt{3}/2)^{2n}}{(2n)!} = \boxed{\cos \frac{\sqrt{3}}{2} - \frac{5}{8}}$$

Problem 7. (20 points)

a) Find the Maclaurin series for e^{-x^2}

$$e^{-x^2} = \sum_{n=0}^{\infty} \frac{(-x^2)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!}$$

b) Find the Maclaurin series for $\int e^{-x^2}\, dx$

$$\int e^{-x^2} dx = \int \left(\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!} \right) dx = \sum_{n=0}^{\infty} \int \frac{(-1)^n x^{2n}}{n!} dx$$

$$= C + \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)\, n!}$$

c) Find $\int_0^{1/2} e^{-x^2}\, dx$ as an infinite series.

$$\int_0^{1/2} e^{-x^2} dx = \left[\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)\, n!} \right]_0^{1/2} = \sum_{n=0}^{\infty} \frac{(-1)^n (1/2)^{2n+1}}{(2n+1)\, n!}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{2n+1} \cdot (2n+1) \cdot n!}$$

d) How many terms of the infinite series from part c) need to be added to approximate the integral accurately to within 10^{-6}? Use the Alternating Series Error Estimation theorem to justify. (You do not need to calculate the approximation. You may use $2^{10} = 1024$.)

$$b_{n+1} = \frac{1}{2^{2(n+1)} (2(n+1)+1)\, (n+1)!} = \frac{1}{2^{2n+3} (2n+3)\, (n+1)!} < \frac{1}{1{,}000{,}000}$$

$n=4$:

$$\frac{1}{2^{11}\, (11)\, 5!} < \frac{1}{1{,}000{,}000}$$

So, $\boxed{\text{5 terms}}$

Problem 8. (10 points) Find the Taylor polynomial $T_3(x)$ centered at $a = 4$ for the function

$$f(x) = \sqrt{2x+1}$$

$T_3(x) = \sum_{i=0}^{3} \frac{f^{(i)}(4)}{i!}(x-4)^i$

$f(x) = \sqrt{2x+1} \rightarrow f(4) = 3$

$f'(x) = \frac{1}{\sqrt{2x+1}} \rightarrow f'(4) = 1/3$

$f''(x) = \frac{-1}{(2x+1)^{3/2}} \rightarrow f''(4) = -1/3^3$

$f'''(x) = \frac{3}{(2x+1)^{5/2}} \rightarrow f'''(4) = 3/3^5$

$T_3(x) = f(4) + f'(4)(x-4) + \frac{f''(4)}{2}(x-4)^2 + \frac{f'''(4)}{3!}(x-4)^3$

$= 3 + \frac{1}{3}(x-4) - \frac{1/27}{2}(x-4)^2 + \frac{1/3^4}{3!}(x-4)^3$

$= 3 + \frac{1}{3}(x-4) - \frac{1}{54}(x-4)^2 + \frac{1}{486}(x-4)^3$

Problem 9. (10 points)

a) Write down the linear approximation to the function $f(x) = \sqrt{x}$ at $a = 9$.

$\boxed{f'(x) = \frac{1}{2\sqrt{x}}}$

The linear approximation is equivalent to $T_1(x)$.

$L(x) = T_1(x) = f(9) + f'(9)(x-9)$

$= 3 + \frac{1}{6}(x-9)$

b) Approximate $\sqrt{11}$ using this approximation.

$\sqrt{x} \simeq 3 + \frac{1}{6}(x-9)$

$\sqrt{11} \simeq 3 + \frac{1}{6}(11-9) = \boxed{\frac{10}{3}}$

c) Use the Taylor's Inequality with the best possible constant M to estimate the error of this approximation.

$\left|\sqrt{11} - \frac{10}{3}\right| = |R_1| \leq \frac{M}{2!}|x-9|^2$ where $9 \leq x \leq 11$

$|f''(x)| = \left|\frac{-1}{4x^{3/2}}\right| \leq M$

$M = \frac{1}{4(9)^{3/2}}$

$= \frac{1}{108}$

$|R_1| \leq \frac{1/108}{2}(11-9)^2 = \boxed{\frac{1}{54}}$

Problem 1. (10 points) Find the interval of convergence of the power series

$$\sum_{n=2}^{\infty} \frac{3^n}{(n^2-n)}(2x-1)^n$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$$

$$= \lim_{n\to\infty} \frac{3^{n+1}|2x-1|^{n+1}}{(n+1)^2-(n+1)} \cdot \frac{n^2-n}{3^n|2x-1|^n}$$

$$= 3|2x-1| \lim_{n\to\infty} \frac{n^2-n}{n^2+n} = 3|2x-1| < 1$$

$$|2x-1| < 1/3$$

$$-1/3 < 2x-1 < 1/3$$

$$2/3 < 2x < 4/3$$

$$1/3 < x < 2/3$$

$x = 1/3$: $\sum \frac{3^n(-1/3)^n}{n^2-n}$

$= \sum \frac{(-1)^n}{n^2-n}$ Convs. by Alt. Series Test

$x = 2/3$: $\sum \frac{3^n(1/3)^n}{n^2-n}$

$= \sum \frac{1}{n^2-n}$ Convs. by limit Comp. test with $\sum \frac{1}{n^2}$.

$$I = [1/3, 2/3]$$

Problem 2. (10 points) Use the power series method to find the limit

$$\lim_{x\to 0} \frac{x - \sin x}{1 + \ln(1+2x) - \cos(2x) - 2x}$$

$$\lim_{x\to 0} \frac{x - \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}}{1 + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}(2x)^n}{n} - \sum_{n=0}^{\infty} \frac{(-1)^n (2x)^{2n}}{(2n)!} - 2x}$$

$$= \lim_{x\to 0} \frac{\cancel{x} - [\cancel{x} - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots]}{\cancel{1} + [\cancel{2x} - \frac{2^2x^2}{\cancel{2}} + \frac{2^3x^3}{3} - \dots] - [\cancel{1} - \frac{2^2x^2}{\cancel{2!}} + \frac{2^4x^4}{4!} - \dots] - \cancel{2x}}$$

$$= \lim_{x\to 0} \frac{\frac{x^3}{3!} - \frac{x^5}{5!} + \dots}{\left[\frac{2^3x^3}{3} - \dots\right] + \left[\frac{-2^4x^4}{4!} + \dots\right]}$$

$$= \lim_{x\to 0} \frac{\frac{1}{3!} - \frac{x^2}{5!} + \dots}{8/3 - \frac{2^4 x}{4!} + \dots}$$

$$= \boxed{1/16}$$

Problem 3. (5 points) Suppose $\lim_{n\to\infty} a_n = 20$. Find the center and radius of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{a_n}{5^n}(x-16)^n$$

Use Ratio Test
(Can also use n^{th} Root test)

$$\lim_{n\to\infty} \frac{|a_{n+1}||x-16|^{n+1}}{5^{n+1}} \cdot \frac{5^n}{|a_n||x-16|^n}$$

$$= \frac{1}{5}|x-16| \lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|}$$

$$= \frac{1}{5}|x-16| \left(\frac{20}{20}\right)$$

$$= \frac{1}{5}|x-16| < 1$$

$$|x-16| < 5$$

$$\boxed{\text{Center @ } a = 16 \text{ and } R = 5}$$

Problem 4. (15 points)
a) Find the Maclaurin series for the function

$$f(x) = \frac{1}{x-1} = \frac{-1}{1-x}$$

$$= -\sum_{n=0}^{\infty} x^n$$

$$= \sum_{n=0}^{\infty} (-1) x^n$$

b) Find the Maclaurin series for the function

$$g(x) = \frac{1}{x-2} = \frac{-1}{2(1-\frac{x}{2})}$$

$$= \frac{-1}{2} \sum_{n=0}^{\infty} \left(\frac{x}{2}\right)^n$$

$$= \sum_{n=0}^{\infty} \frac{-1}{2^{n+1}} x^n$$

c) Find the Maclaurin series for the function

$$h(x) = \frac{1}{(x-1)(x-2)}.$$

(Hint: use partial fractions to relate $h(x)$ to $f(x)$ and $g(x)$.)

$$\frac{1}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}$$

$$1 = A(x-2) + B(x-1)$$

$x=1$: $1 = -A \Rightarrow A = -1$

$x=2$: $1 = B$

so,

$$h(x) = \frac{-1}{x-1} + \frac{1}{x-2}$$

$$= \sum_{n=0}^{\infty} x^n - \sum_{n=0}^{\infty} \frac{1}{2^{n+1}} x^n$$

$$= \sum_{n=0}^{\infty} \left(1 - \frac{1}{2^{n+1}}\right) x^n$$

Problem 5. (10 points)
Find $T_4(x)$ centered at $a = 0$ for the function

$$f(x) = \sin(2x) \cdot \cos(3x)$$

Multiplication of Power Series and Taylor Polynomials will not be covered on final.

Problem 6. (10 points)
a) Write down the quadratic approximation to the function $f(x) = x^{3/2}$ at $a = 4$.

$f'(x) = \frac{3}{2}\sqrt{x} \to f'(4) = 3$

$f''(x) = \frac{3}{4\sqrt{x}} \to f''(4) = 3/8$

$f'''(x) = \frac{-3}{8}\frac{1}{x^{3/2}}$

$T_2(x) = f(4) + f'(4)(x-4) + \frac{f''(4)}{2}(x-4)^2$

$T_2(x) = 8 + 3(x-4) + \frac{3}{16}(x-4)^2$

b) Approximate $(\sqrt{4.4})^3$ using this approximation.

$f(x) = x^{3/2} \simeq T_2(x)$ for x near 4.

$(\sqrt{4.4})^3 = (4.4)^{3/2} \simeq 8 + 3(4.4-4) + \frac{3}{16}(4.4-4)^2$

$= 8 + 3(.4) + 3(.4)^2$

$= \boxed{9.23}$

c) Use Taylor's Inequality with the best possible constant M to estimate the error of this approximation.

$|(4.4)^{3/2} - 9.23| = |R_2| \leq \frac{M}{3!}|x-4|^3$ for $4 \leq x \leq 4.4$

$0 \leq x-4 \leq .4$

$|f'''(x)| = \frac{3}{8}\frac{1}{x^{3/2}} \leq \frac{3}{8}\frac{1}{(4)^{3/2}}$

$= \boxed{3/64 = M}$

$|x-4|^3 \leq (.4)^3 = .064$

$|R_2| \leq \frac{3/64}{3!}(.064)$

$= \frac{3(.001)}{6}$

$= \frac{1}{2000}$

Problem 7. (20 points)

a) Find the Maclaurin series for $x^4 \cos 3x^2$

$$x^4\cos(3x^2) = x^4 \sum_{n=0}^{\infty} \frac{(-1)^n (3x^2)^{2n}}{(2n)!}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n}}{(2n)!} x^{4n+4}$$

b) Find the Maclaurin series for $\int x^4 \cos 3x^2 \, dx$

$$\int x^4\cos(3x^2)\,dx = \int \sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n}}{(2n)!} x^{4n+4}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n}}{(2n)!} \int x^{4n+4} dx = C + \sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n} x^{4n+5}}{(2n)!(4n+5)}$$

c) Find $\int_0^2 x^4 \cos 3x^2 \, dx$ as an infinite series.

$$\int_0^2 x^4\cos(3x^2)\,dx = \left[\sum_{n=0}^{\infty} \frac{(-1)^n \cdot 3^{2n}}{(2n)!(4n+5)} x^{4n+5}\right]_0^2$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n \cdot 3^{2n} \cdot 2^{4n+5}}{(2n)!(4n+5)}$$

Problem 8. (10 points) Find the Taylor polynomial $T_3(x)$ centered at $a = 1$ for the function

$$f(x) = \arctan x \rightarrow f(1) = \arctan(1) = \pi/4$$

$f'(x) = \frac{1}{1+x^2} \rightarrow f'(1) = 1/2$

$f''(x) = \frac{-2x}{(1+x^2)^2} \rightarrow f''(1) = -1/2$

$f'''(x) = \frac{-2}{(1+x^2)^2} + \frac{8x^2}{(1+x^2)^3} \rightarrow f'''(1) = 1/2$

$T_3(x) = f(1) + f'(1)(x-1) + \frac{f''(1)}{2!}(x-1)^2 + \frac{f'''(1)}{3!}(x-1)^3$

$= \frac{\pi}{4} + \frac{1}{2}(x-1) - \frac{1}{4}(x-1)^2 + \frac{1}{12}(x-1)^3$

Problem 9. (10 points) Find the sum of the series.

a) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$ $= \arctan(1) = \pi/4$

b) $\sum_{n=2}^{\infty} \frac{3^n}{n!}$

$\sum_{n=0}^{\infty} \frac{3^n}{n!} = e^3$

$1 + 3 + \sum_{n=2}^{\infty} \frac{3^n}{n!} = e^3$

$\sum_{n=2}^{\infty} \frac{3^n}{n!} = \boxed{e^3 - 4}$

Problem 5. $T_4(x) = 2x - \frac{23}{3}x^3$

Problem 6.

a) $T_2(x) = 8 + 3(x-4) + \frac{3}{16}(x-4)^2$

b) 9.23

c) $|4.4^{3/2} - 9.23| = |R_2| \leq \frac{1}{2000}$ (or 0.0005)

Problem 7.

a) $\sum\limits_{n=0}^{\infty} \frac{(-1)^n \cdot 3^{2n}}{(2n)!} x^{4n+4}$

b) $C + \sum\limits_{n=0}^{\infty} \frac{(-1)^n \cdot 3^{2n}}{(4n+5)(2n)!} x^{4n+5}$

c) $\sum\limits_{n=0}^{\infty} \frac{(-1)^n \cdot 3^{2n} \cdot 2^{4n+5}}{(4n+5)(2n)!}$

Problem 8. $T_3(x) = \frac{\pi}{4} + \frac{1}{2}(x-1) - \frac{1}{4}(x-1)^2$

Problem 9.

a) $\frac{\pi}{4}$

b) $e^3 - 4$

Math 227 Sample Final Examination 3 Answers

Problem 1. $(-2, 1)$

Problem 2. 1

Problem 3. Center is $-\frac{7}{2}$. Radius is 5.

Problem 4.

a) $\sum_{n=0}^{\infty}(-1)^n x^n$

b) $\frac{1}{(x+1)^2} = \sum_{n=1}^{\infty}(-1)^{n-1} n x^{n-1}$ (or $\sum_{n=0}^{\infty}(-1)^n (n+1) x^n$)

c) $\frac{1}{(x+1)^3} = \sum_{n=2}^{\infty} \frac{1}{2}(-1)^n n(n-1) x^{n-2}$

d) $\frac{7}{2}$

Problem 5. $e + 2e(x-1) + 3e(x-1)^2 + \frac{10e}{3}(x-1)^3$

Problem 6.

a) $e^{0.1} = \sum_{n=0}^{\infty} \frac{1}{10^n \cdot n!}$

b) $T_2(0.1) = 1.105$ It is the 3rd partial sum of the series from part a).

c) $n = 4$

Problem 7. (15 points)

a) $x \arctan(x^3) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{6n+4}$

b) $\int x \arctan(x^3)\ dx = C + \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)(6n+5)} x^{6n+5}$

c) $\int_0^{1/2} x \arctan(x^3)\ dx = C + \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)(6n+5)2^{6n+5}}$

Problem 8.

a) 0

b) $1 - \ln 2$