Math 225 Midterm Examination Fall 2015, 8:00 am version

Name (print) Key

Name (sign)______________________

Bing ID number ______________________
(Your instructor may check your ID during or after the test)

No books, no notes, no electronic devices (calculators, cell phones, smart watches, etc.) Write all your work on the test – nothing else will be graded. **You must show all your work.** Your work must be legible, and the final answers must be reasonably simplified.

On some problems you may be asked to use a specific method to solve the problem (for instance, "Use the Fundamental Theorem of Calculus to find..."). On all other problems, you may use any method we have covered. **You may not use methods that we have not covered.**

Wandering Eyes Policy

You must keep your eyes on your own work at all times. If you are found looking around, you will be warned once, and only once. A second infraction may result in automatic zero on this test, and possibly a referral to the Harpur College Academic Honesty Committee.

Duration of the Test

This is a timed test designed for one class period. You will start the test when your proctor tells you to start, and you will finish the test when your proctor tells you to stop, when the class period is over.

1. (10 points) Find the most general antiderivative.

a)

$$\int (\sec^2 x) \cdot \sqrt{2\tan x + 5}\, dx = \int \sec^2 x \sqrt{u} \cdot \frac{du}{2\sec^2 x} =$$

$u = 2\tan x + 5$

$du = 2\sec^2 x\, dx$

$dx = \frac{du}{2\sec^2 x}$

$$= \int \frac{1}{2}\sqrt{u}\, du$$

$$= \frac{1}{2} \cdot u^{3/2} \cdot \frac{2}{3} + C$$

$$= \frac{1}{3}(2\tan x + 5)^{3/2} + C$$

b)

$$\int \frac{x}{(x^2+4)^3}\, dx = \frac{1}{2}\int \frac{2x\, dx}{u^3} =$$

$u = x^2 + 4$

$du = 2x\, dx$

$$= \frac{1}{2}\int \frac{du}{u^3} = \frac{1}{2} u^{-2} \cdot \frac{1}{-2} + C$$

$$= -\frac{1}{4}(x^2+4)^{-2} + C$$

$$\left(\text{or } -\frac{1}{4(x^2+4)^2} + C \right)$$

2. (10 points) Find the derivative.

$$\frac{d}{dx}\int_{\sqrt{x}}^{\cos x}\frac{\sqrt{t^3+1}}{t+7}\,dt =$$

$$= \frac{\sqrt{\cos^3 x+1}}{\cos x+7}\cdot(-\sin x) - \frac{\sqrt{(\sqrt{x})^3+1}}{\sqrt{x}+7}\cdot\frac{1}{2\sqrt{x}}$$

3. (10 points) A particle moves along a straight line so that the velocity at time t is $v(t) = 2t - 6$ (measured in m/s). Find the total distance traveled during the time period $0 \leq t \leq 4$.

1st way. $v(t) \leq 0$ for $0 \leq t \leq 3$,
$v(t) \geq 0$ for $3 \leq t \leq 4$

$p(t) = t^2 - 6t + C$

Can assume $p(0) = 0$, so $C = 0$.

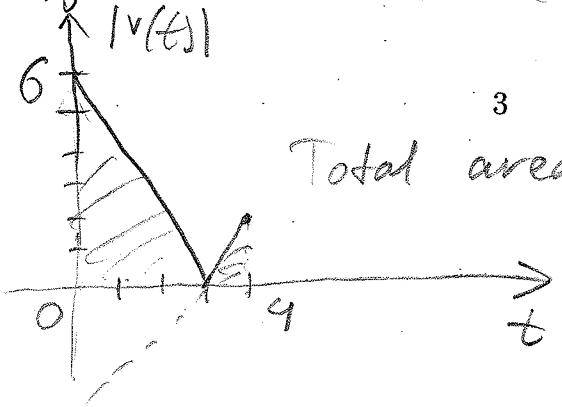
Then $p(0) = 0$, $p(3) = 9 - 18 = -9$, $p(4) = 16 - 24 = -8$.

Total distance: $9 + 1 = 10$

2nd way $\int_0^4 |2t-6|\,dt = \int_0^3 (6-2t)\,dt + \int_3^4 (2t-6)\,dt$

$= (6t - t^2)\Big|_0^3 + (t^2 - 6t)\Big|_3^4 = (9-0) + (-8+9) = 10$

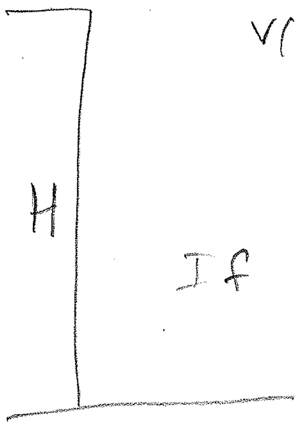
3rd way



Total area $= \frac{1}{2}\cdot 3\cdot 6 + \frac{1}{2}\cdot 1\cdot 2 = 10$

4. (15 points)A rock is dropped from a cliff and hits the surface of the lake below 3 seconds later. Find the distance between the rock and the surface of the lake as a function of time t (in seconds), starting from the moment it was dropped.

NOTE: Assume that the acceleration of gravity is 10 m/s^2 and ignore the drag.



$v(t) = -10t + C$

$v(0) = 0 \quad 0 = -10 \cdot 0 + C \quad C = 0$

$\boxed{v(t) = -10t}$

$h(t) = -5t^2 + D$

If the height of the cliff is H,

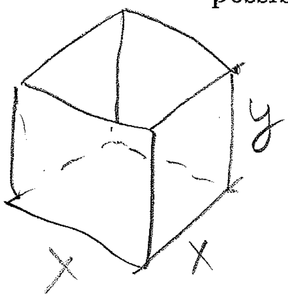
$H = -5 \cdot 0^2 + D,$

$\boxed{h(t) = -5t^2 + H}$

$h(3) = 0 \quad 0 = -5 \cdot 3^2 + H \quad H = 45$

$h(t) = -5t^2 + 45$

5. (15 points) We are planning to make a box with an open top and square base. It has to have the volume of 6 ft^3. The material for the base costs \$3.00 per square foot and the material for the sides costs \$2.00 per square foot. Find the dimensions of such box that result in the smallest possible total cost of the material. Justify.



$$x^2 y = 6$$

$$C = 3 \cdot x^2 + 2 \cdot 4xy = 3x^2 + 8xy$$

$$y = \frac{6}{x^2}$$

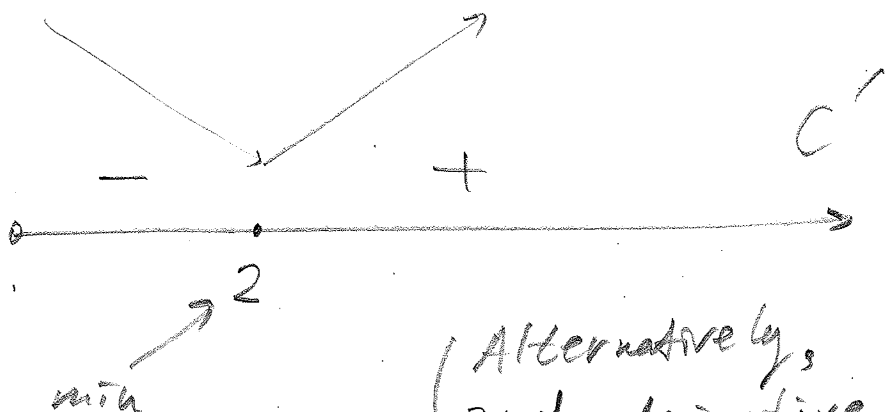
$$C(x) = 3x^2 + 8x\frac{6}{x^2} = 3x^2 + 48 \cdot \frac{1}{x}$$

$$C'(x) = 6x - 48 \cdot \frac{1}{x^2} = \frac{6x^3 - 48}{x^2} = \frac{6(x^3-8)}{x^2}$$

$x^3 - 8 = 0$

$x = 2$

$x > 0$



(Alternatively, 2nd derivative test may be used)

$$y = \frac{6}{2^2} = \frac{3}{2}$$

$$\boxed{2\text{ ft} \times 2\text{ ft} \times \frac{3}{2}\text{ ft}}$$

6. (15 points) Find $F(x)$ for the following:

a)

$$F'(x) = 3\sin x + 2\cos x - 5x^3, \quad F(0) = 1$$

$$F(x) = -3\cos x + 2\sin x - \frac{5}{4}x^4 + C$$

$x=0$: $\quad 1 = -3 + 0 - 0 + C \qquad C = 4$

$$F(x) = -3\cos x + 2\sin x - \frac{5}{4}x^4 + 4$$

b)

$$F'(x) = x \cdot (3x+1)^7, \quad F(0) = 2$$

$$\int x(3x+1)^7 dx = \int \frac{u-1}{3} u^7 \cdot \frac{du}{3} = \frac{1}{9}\int (u^8 - u^7)\,du$$

$u = 3x+1 \qquad x = \frac{u-1}{3}$

$du = 3\,dx$

$$= \frac{1}{9}\left(\frac{u^9}{9} - \frac{u^8}{8}\right) + C$$

$$= \frac{1}{9}\left(\frac{(3x+1)^9}{9} - \frac{(3x+1)^8}{8}\right) + C$$

$F(0)=2$: $\quad \frac{8}{72} - \frac{9}{72} = -\frac{1}{72}$

$$2 = \frac{1}{9}\left(\frac{1}{9} - \frac{1}{8}\right) + C \qquad C = 2 + \frac{1}{9}\cdot\frac{1}{72} = 2\frac{1}{648}$$

$$F(x) = \frac{1}{9}\left(\frac{(3x+1)^9}{9} - \frac{(3x+1)^8}{8}\right) + 2\frac{1}{648}$$

or $\left(2 - \frac{1}{81} + \frac{1}{72}\right)$

7. (10 points)

a) Evaluate the definite integral.

$$\int_0^2 \frac{x^2}{\sqrt{x^3+1}}\,dx = \int_{x=0}^{x=2} \frac{x^2}{\sqrt{u}}\,\frac{du}{3x^2} = \int_1^9 \frac{du}{3\sqrt{u}}$$

$u = x^3+1$ $\quad dx = \frac{du}{3x^2}$

$du = 3x^2 dx$

$$= \frac{1}{3} u^{1/2}/(1/2) \Big|_1^9$$

$$= \frac{2}{3} u^{1/2} \Big|_1^9 = \frac{2}{3}(3-1) = \frac{4}{3}$$

b) Given that f(x) is continuous and $\int_3^{12} f(x)\,dx = 5$, find $\int_1^4 f(3x)\,dx$.

$$\int_1^4 f(3x)\,dx = \int_3^{12} f(u)\cdot\frac{1}{3}\,du = \frac{1}{3}\cdot 5 = \frac{5}{3}$$

$u = 3x$

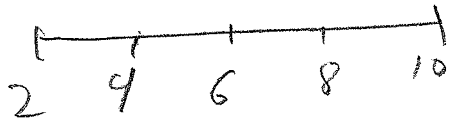
$du = 3dx$

8. (15 points)
a) Calculate the Left-point approximation L_n to integral

$$\int_2^{10} \frac{1}{x}\, dx$$

with $n = 4$.

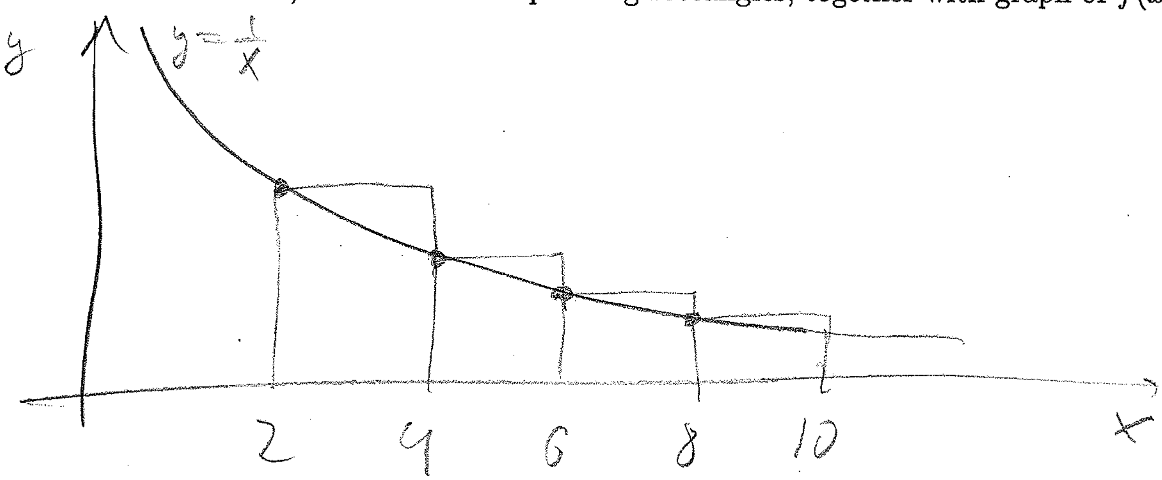
$$\Delta x = \frac{10-2}{4} = 2$$



$$L_4 = 2\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8}\right)$$

$$= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{3}{2} + \frac{7}{12} = \frac{25}{12}$$

b) Draw the corresponding rectangles, together with graph of $f(x) = \frac{1}{x}$.



c) Is L_4 is greater than the actual integral or less than it? Specify and explain, or explain why this cannot be determined.

L_4 is greater than the integral, because the total area of rectangles from part b) is greater than the area below the graph of f(x)