Math 224: Exam 2 **Name** ___________________________________

Spring 2016 **Instructor** ________________________________

Problem	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**Total**	**Course Points**
Points	**10**	**10**	**10**	**15**	**10**	**10**	**10**	**25**	**100**	**300**
Score										

- Calculators and/or other electronic devices are not permitted for this test.
- Show your work unless the problem requires only a short answer. You may not use L'Hopital's Rule to evaluate limits on this exam.
- There are problems on the front of all pages. If you need more room, use the backs of the pages for scrap paper.
- It is highly advised that you complete this exam in pencil rather than pen.
- You have exactly 90 minutes, starting from the beginning of class, to complete this exam. When the time is up, stop writing and turn in the exam to your instructor. Students who continue writing after time is called may be penalized.

1. (10 points)

Find the equation of the tangent line to the curve $4x^2 + 2xy - xy^3 = 0$ at the point $(1, 2)$.

2. (10 points)

Fred tosses a pebble into a pond at exactly 12:00 PM. Immediately, a circular ripple spreads out from the impact, and the radius of the ripple grows at a rate of 0.25 feet per second. What is the rate of growth of the area inside the ripple at exactly 12:01 PM?

3. (10 points)

A 13-foot ladder is leaning toward against a wall and sliding downward toward the floor such that the distance between the base of the ladder and the wall is increasing by 3 feet per second. How fast is the angle between the floor and the ladder changing when the distance between the base of the ladder and the wall is 12 feet?

4. (15 points)

A drone's height (in feet above the ground) is given as a function of time (in seconds) by $h(t) = -27t + 18t^2 - 3t^3$, where $0 \leq t \leq 4$. Answer the following questions and include proper units as part of your answer.

a) What is the velocity of the drone when $t = 2$?

b) What is the acceleration of the drone when $t = 2$?

c) On what time interval(s) is the drone moving downward? Justify your answer.

d) Find the total distance traveled by the drone in the time interval from $t = 0$ to $t = 4$.

5. (10 points)

a) Find the linearization $L(x)$ of the function $f(x) = x^{3/4}$ when $a = 16$.

b) Use your answer from part a) to estimate the value of $15^{3/4}$.

6. (10 points)

Let $g(x) = x - 3x^{1/3}$.

Find the absolute maximum and the absolute minimum of g on the interval $[-1, 2]$.

7. (10 points)

Evaluate each of the following limits. Show all necessary work.

a) $\lim\limits_{x \to -\infty} \frac{\sqrt{9x^2+1}}{x-4}$

b) $\lim\limits_{x \to \infty} \left(x - \sqrt{x^2 + x} \right)$

8. (25 points)

Suppose $f(x) = \frac{2x}{(x-1)^2}$

Then $f'(x) = \frac{-2(x+1)}{(x-1)^3}$ and $f''(x) = \frac{4(x+2)}{(x-1)^4}$.

a) Find the domain of f.

b) Indicate the intervals where f satisfies the given condition. Support your answers.

f is increasing on ________________________

f is decreasing on ________________________

f is concave upward on _______________________

f is concave downward on _______________________

c) Find ordered pairs for each of the following. Write "none" if the function does not have the specified characteristic. Provide support for your answers.

f has a local minimum at _________________

f has a local maximum at _________________

f has points of inflection at _________________

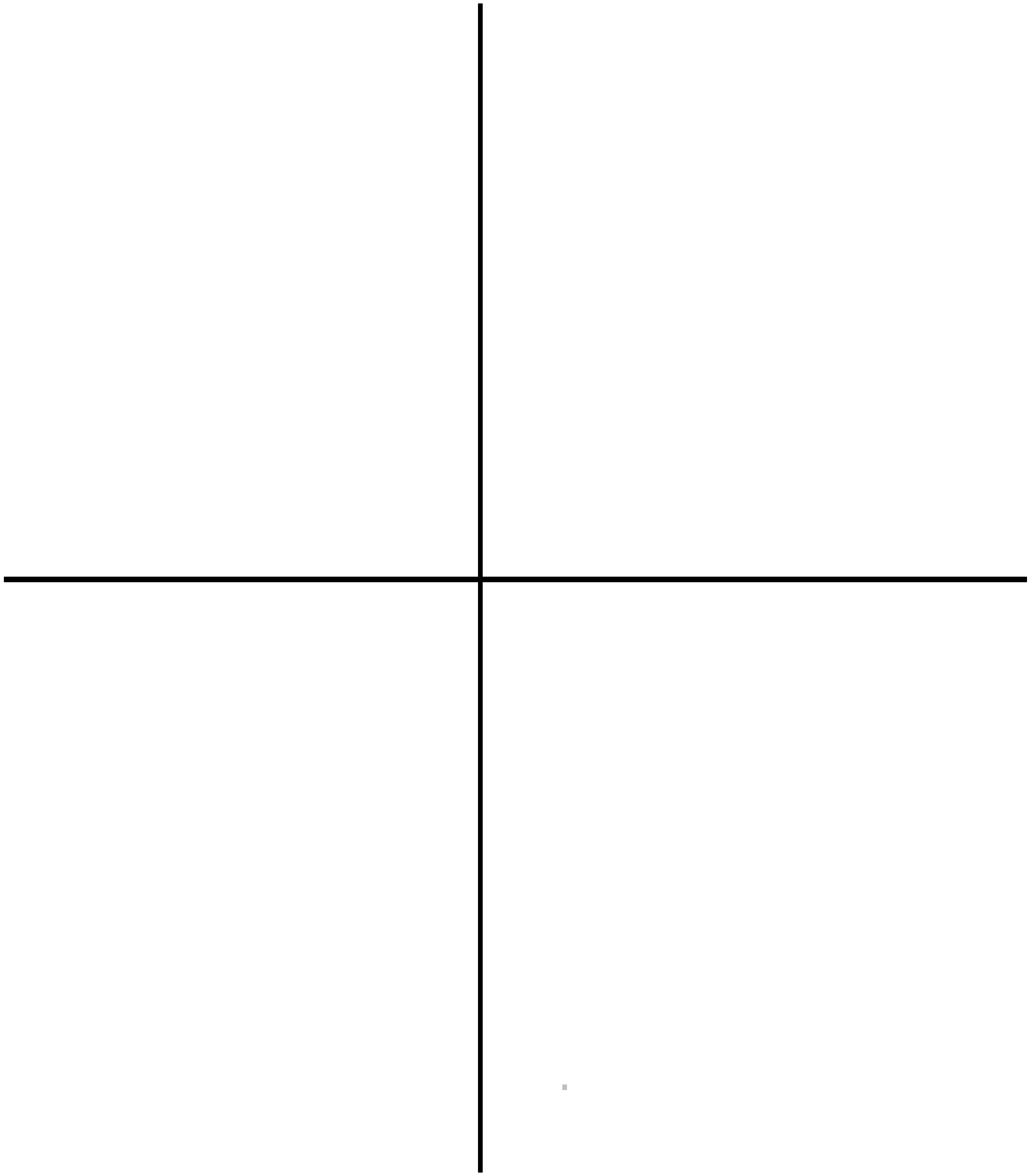
d) Give the equations of all asymptotes. Write "none" if the function does not have any asymptotes of the specified type. Use appropriate limits to justify your answer.

f has horizontal asymptote(s) ________________

f has vertical asymptote(s) ________________

e) On the next page, use the information found above to sketch the graph of f. Be sure to label your axes appropriately.

GRAPH FOR PROBLEM #7



1. (10 points)

Find the equation of the tangent line to the curve $4x^2 + 2xy - xy^3 = 0$ at the point $(1, 2)$.

$$8x + 2x \cdot \frac{dy}{dx} + 2y - \left(x \cdot 3y^2 \frac{dy}{dx} + y^3\right) = 0$$

$$8x + 2x\frac{dy}{dx} + 2y - 3xy^2\frac{dy}{dx} - y^3 = 0$$

$$2x\frac{dy}{dx} - 3xy^2\frac{dy}{dx} = y^3 - 2y - 8x$$

$$\frac{dy}{dx}(2x - 3xy^2) = y^3 - 2y - 8x$$

$$\frac{dy}{dx} = \frac{y^3 - 2y - 8x}{2x - 3xy^2}$$

$$\left.\frac{dy}{dx}\right|_{(1,2)} = \frac{(2)^3 - 2(2) - 8(1)}{2(1) - 3(1)(2)^2}$$

$$= \frac{8 - 4 - 8}{2 - 12} = \frac{-4}{-10} = \frac{2}{5}$$

tangent line at $(1,2)$ is $y - 2 = \frac{2}{5}(x - 1)$

2. (10 points)

Fred tosses a pebble into a pond at exactly 12:00 PM. Immediately, a circular ripple spreads out from the impact, and the radius of the ripple grows at a rate of 0.25 feet per second. What is the rate of growth of the area inside the ripple at exactly 12:01 PM?

KNOWN: $\frac{dr}{dt} = 0.25$ ft/s.

FIND: $\frac{dA}{dt}$ after 60 seconds (1 minute) has elapsed.

$$A = \pi r^2$$

After 60 seconds,
$r = 0.25(60) = 15$ ft.

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi(15)(.25)$$

$$= \frac{30\pi}{4}$$

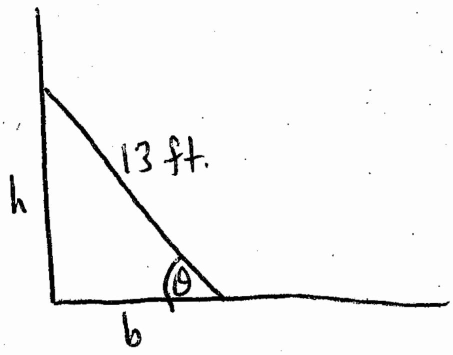
$$= \frac{15\pi}{2} \text{ ft}^2/\text{s}$$

3. (10 points)

A 13-foot ladder is leaning toward against a wall and sliding downward toward the floor such that the distance between the base of the ladder and the wall is increasing by 3 feet per second. How fast is the angle between the floor and the ladder changing when the distance between the base of the ladder and the wall is 12 feet?

KNOWN: $\frac{db}{dt} = 3$ ft/s

FIND: $\frac{d\theta}{dt}$ when $b = 12$ feet.



$\cos(\theta) = \frac{1}{13} b$

$-\sin(\theta)\frac{d\theta}{dt} = \frac{1}{13}\frac{db}{dt}$

when $b = 12$, $h = 5$

So $\sin(\theta) = \frac{5}{13}$

$-\frac{5}{13}\frac{d\theta}{dt} = \frac{1}{13}(3)$

$-5\frac{d\theta}{dt} = 3$

$\frac{d\theta}{dt} = -\frac{3}{5}$ radians/s.

θ is decreasing by $\frac{3}{5}$ radians/s when the base of the ladder is 12 feet from the wall.

4. (15 points)

A drone's height (in feet above the ground) is given as a function of time (in seconds) by $h(t) = -27t + 18t^2 - 3t^3$, where $0 \le t \le 4$. Answer the following questions and include proper units as part of your answer.

a) What is the velocity of the drone when $t = 2$?

$$v(t) = h'(t) = -27 + 36t - 9t^2$$

$$v(2) = -27 + 36(2) - 9(2)^2$$

$$= -27 + 72 - 36 = 9 \text{ ft/sec.}$$

b) What is the acceleration of the drone when $t = 2$?

$$a(t) = v'(t) = h''(t) = 36 - 18t$$

$$a(2) = 36 - 18(2) = 0 \text{ ft/sec}^2$$

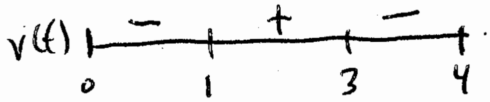
c) On what time interval(s) is the drone moving downward? Justify your answer.

moving downward $\Rightarrow v(t) < 0$

$$-9t^2 + 36t - 27 < 0$$

$$-9(t^2 - 4t + 3) < 0$$

$$-9(t-3)(t-1) < 0$$



The drone is moving downward on $0 < t < 1$ and $3 < t < 4$.

d) Find the total distance traveled by the drone in the time interval from $t = 0$ to $t = 4$.

Total Distance $= |h(1) - h(0)| + |h(3) - h(1)| + |h(4) - h(3)|$

$$= |-12 - 0| + |0 - (-12)| + |-12 - 0|$$

$$= |-12| + |12| + |-12|$$

$$= 12 + 12 + 12$$

$$= 36 \text{ feet.}$$

$h(0) = 0$

$h(1) = -27 + 18 - 3 = -12$

$h(3) = -27(3) + 18(3)^2 - 3(3)^3 = -81 + 162 - 81 = 0$

$h(4) = -27(4) + 18(4)^2 - 3(4)^3 = -108 + 288 - 192 = -12$

5. (10 points)

a) Find the linearization $L(x)$ of the function $f(x) = x^{3/4}$ when $a = 16$.

$f(16) = 16^{3/4} = (\sqrt[4]{16})^3 = 2^3 = 8$ $(16, 8)$ point.

$f'(x) = \frac{3}{4}x^{-1/4} = \frac{3}{4\sqrt[4]{x}}$

$f'(16) = \frac{3}{4\sqrt[4]{16}} = \frac{3}{4 \cdot 2} = \frac{3}{8}$ slope

$L(x) = \frac{3}{8}(x-16) + 8$

b) Use your answer from part a) to estimate the value of $15^{3/4}$.

$L(15) = \frac{3}{8}(15-16) + 8$

$= -\frac{3}{8} + 8$

$= \frac{61}{8}$

6. (10 points)

Let $g(x) = x - 3x^{1/3}$.

Find the absolute maximum and the absolute minimum of g on the interval $[-1, 2]$.

$g'(x) = 1 - x^{-2/3}$

g' DNE when $x = 0$, so $x = 0$ is a critical number.

$g' = 0$? $\quad 0 = 1 - x^{-2/3}$

$$\Rightarrow x^{-2/3} = 1$$

$$\frac{1}{x^{2/3}} = 1$$

$$x^{2/3} = 1$$

$\Rightarrow x = \pm 1$ so $x = \pm 1$ are critical numbers

Check endpoints and critical numbers

x	$g(x)$
-1	2
0	0
1	-2
2	$2-3\sqrt[3]{2}$

$g(-1) = -1 - 3(-1)^{1/3}$

$= -1 - 3(-1) = -1 + 3 = 2$

$g(0) = 0 - 3(0)^{1/3} = 0$

$g(1) = 1 - 3(1)^{1/3} = 1 - 3 = -2$

$g(2) = 2 - 3(2)^{1/3}$

$= 2 - 3\sqrt[3]{2}$

Absolute maximum on $[-1, 2]$ is 2

Absolute minimum on $[-1, 2]$ is -2

7. (10 points)

Evaluate each of the following limits. Show all necessary work.

a) $\lim\limits_{x\to-\infty} \frac{\sqrt{9x^2+1}}{x-4}$

$$= \lim_{x\to-\infty} \frac{\frac{\sqrt{9x^2+1}}{x}}{\frac{x}{x}-\frac{4}{x}}$$

$$\sqrt{x^2} = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

So as $x \to -\infty$,
$\sqrt{x^2} = -x \Rightarrow x = -\sqrt{x^2}$

$$= \lim_{x\to-\infty} \frac{\frac{\sqrt{9x^2+1}}{-\sqrt{x^2}}}{1-\frac{4}{x}} = \lim_{x\to-\infty} \frac{-\sqrt{\frac{9x^2}{x^2}+\frac{1}{x^2}}}{1-\frac{4}{x}} = \lim_{x\to-\infty} \frac{-\sqrt{9+\frac{1}{x^2}}}{1-\frac{4}{x}}$$

$$= \frac{-\sqrt{9}}{1} = -3$$

b) $\lim\limits_{x\to\infty}(x-\sqrt{x^2+x})$

$$\lim_{x\to\infty} \frac{(x-\sqrt{x^2+x})(x+\sqrt{x^2+x})}{(x+\sqrt{x^2+x})}$$

$$= \lim_{x\to\infty} \frac{x^2-(x^2+x)}{x+\sqrt{x^2+x}}$$

$$= \lim_{x\to\infty} \frac{-x}{x+\sqrt{x^2+x}}$$

$$= \lim_{x\to\infty} \frac{-\frac{x}{x}}{\frac{x}{x}+\frac{\sqrt{x^2+x}}{x}} = \lim_{x\to\infty} \frac{-1}{1+\sqrt{\frac{x^2+x}{x^2}}}$$

$$= \lim_{x\to\infty} \frac{-1}{1+\sqrt{1+\frac{1}{x}}} = \frac{-1}{1+\sqrt{1}}$$

$$= -\frac{1}{2}$$

8. (25 points)

Suppose $f(x) = \frac{2x}{(x-1)^2}$

Then $f'(x) = \frac{-2(x+1)}{(x-1)^3}$ and $f''(x) = \frac{4(x+2)}{(x-1)^4}$.

a) Find the domain of f. $\{x \mid x \neq 1\}$

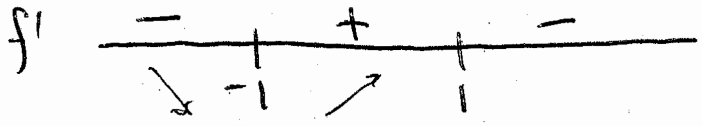
b) Indicate the intervals where f satisfies the given condition. Support your answers in the space below.

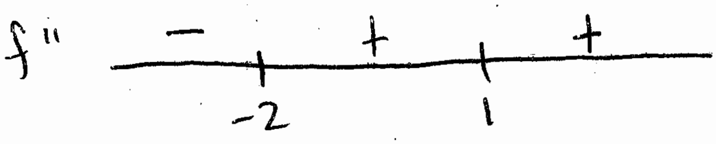
f is increasing on $(-1, 1)$

f is decreasing on $(-\infty, -1) \cup (1, \infty)$

f is concave upward on $(-2, 1) \cup (1, \infty)$

f is concave downward on $(-\infty, -2)$





c) Find ordered pairs for each of the following. Write "none" if the function does not have the specified characteristic. Provide support for your answers in the space below.

f has a local minimum at $\left(-1, -\frac{1}{2}\right)$

f has a local maximum at none

f has points of inflection at $\left(-2, -\frac{4}{9}\right)$

At $x = -1$ f switches from decreasing to increasing so there is a local minimum

$$f(-1) = \frac{2(-1)}{(-1-1)^2} = \frac{-2}{4} = -\frac{1}{2}$$

At $x = -2$ f is defined and f'' changes sign. So there is an inflection point.

$$f(-2) = \frac{2(-2)}{(-2-1)^2} = \frac{-4}{9}$$

d) Give the equations of all asymptotes. Write "none" if the function does not have any asymptotes of the specified type. Use appropriate limits to justify your answer in the space below.

f has horizontal asymptote(s) $y = 0$

f has vertical asymptote(s) $X = 1$

$\lim_{X \to 1^-} \frac{2X}{(X-1)^2} = \infty$ and $\lim_{X \to 1^+} \frac{2X}{(X-1)^2} = \infty$ so $X = 1$ is a vertical Asymptote.

$$\lim_{X \to \pm\infty} \frac{2x}{(X-1)^2} = \lim_{X \to \pm\infty} \frac{2x}{X^2-2x+1} = \lim_{X \to \pm\infty} \frac{\frac{2x}{x^2}}{\frac{x^2}{x^2} - \frac{2x}{x^2} + \frac{1}{x^2}}$$

$$= \lim_{X \to \pm\infty} \frac{\frac{2}{x}}{1 - \frac{2}{x} + \frac{1}{x^2}} = \frac{0}{1} = 0$$

$\Rightarrow$ $y = 0$ is a HORIZONTAL ASYMPTOTE.

e) On the next page, use the information found in parts a) through d) to sketch the graph of f. Be sure to label your axes appropriately.

GRAPH FOR PROBLEM #8