Math 224: Exam 1

Spring 2016

Name ANSWERS

Instructor ____________________

Problem	1	2	3	4	5	6	7	8	Total	Course Points
Points	12	8	20	11	15	10	12	12	100	300
Score										

- Do not open this test until your instructor tells you to begin.

- Calculators and/or other electronic devices are not permitted for this test.

- Show your work unless the problem requires only a short answer.

- There are problems on the front of all pages. If you need more room, use the backs of the pages for scrap paper.

- It is highly advised that you complete this exam in pencil rather than pen.

- You have exactly 90 minutes, starting from the beginning of class, to complete this exam.

- When the time is up, stop writing and turn in the exam to your instructor. Students who continue writing after time is called may be penalized.

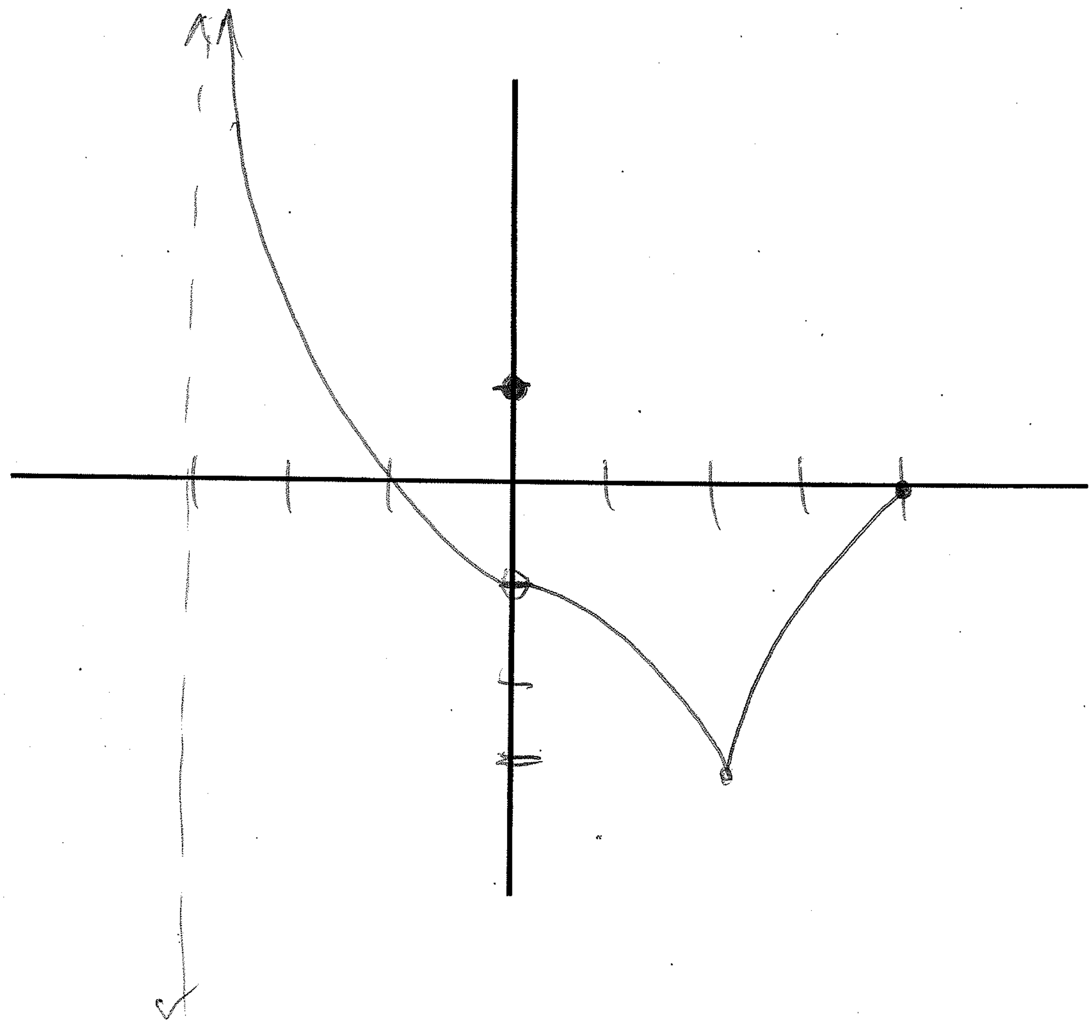
- If you finish early, turn in the exam to your instructor and exit quietly.

- Good luck!

1. (12 points) Sketch the graph of a function f with domain (-3, 4] satisfying all six of the following criteria. Use the axes below. Be sure to scale the axes appropriately.

a) $\lim_{x \to -3^+} f(x) = \infty$ b) $\lim_{x \to 0} f(x) = -1$ c) $f(0) = 1$

d) $\lim_{x \to 2} f(x) = -3$ e) $f'(2)$ does not exist. f) $f(4) = 0$



2. (8 points) Suppose $f(x) = \frac{2x+8}{x^2+2x-8}$.

Find the equations of all vertical asymptotes to the graph of f. Use appropriate limits to justify your answer.

$$f(x) = \frac{2x+8}{x^2+2x-8} = \frac{2x+8}{(x+4)(x-2)}$$

Need to check $x = -4$, $x = 2$.

$$\lim_{x \to -4} f(x) = \lim_{x \to -4} \frac{2x+8}{(x+4)(x-2)} = \lim_{x \to -4} \frac{2(x+4)}{(x+4)(x-2)}$$

$$= \lim_{x \to -4} \frac{2}{x-2} = \frac{2}{-6} = -\frac{1}{3}$$

Thus there is no Vertical Asymptote at $x = -4$.

$$\lim_{x \to 2} f(x) = \lim_{x \to 2} \frac{2x+8}{(x+4)(x-2)} = \lim_{x \to 2} \frac{2}{x-2} \Rightarrow$$ Need to check left/right hand limits.

$$\left.\begin{aligned} \lim_{x \to 2^-} \frac{2}{x-2} &= -\infty \\ \lim_{x \to 2^+} \frac{2}{x-2} &= \infty \end{aligned}\right\}$$ Thus $x = 2$ is a Vertical Asymptote.

3. (20 points) Evaluate each of the following limits, if they exist. If your answer is "DNE," be sure to fully explain why.

a) $\lim\limits_{x\to\frac{1}{2}} \frac{\sqrt{2x+3}-2}{2x-1} = \lim\limits_{x\to\frac{1}{2}} \frac{(\sqrt{2x+3}-2)(\sqrt{2x+3}+2)}{(2x-1)(\sqrt{2x+3}+2)}$

$= \lim\limits_{x\to\frac{1}{2}} \frac{2x+3-4}{(2x-1)(\sqrt{2x+3}+2)} = \lim\limits_{x\to 1} \frac{2x-1}{(2x-1)(\sqrt{2x+3}+2)}$

$= \frac{1}{\sqrt{2(\frac{1}{2})+3}+2} = \frac{1}{\sqrt{4}+2} = \frac{1}{4}$

b) $\lim\limits_{x\to 4^-} \frac{|x-4|}{3x^2-3x-36} = \lim\limits_{x\to 4^-} \frac{-(x-4)}{3(x^2-x-12)} = \lim\limits_{x\to 4^-} \frac{-(x-4)}{3(x-4)(x+3)}$

$= \lim\limits_{x\to 4^-} \frac{-1}{3(x+3)}$

$= \frac{-1}{3(4+3)} = -\frac{1}{21}$

c) $\lim\limits_{x\to 0} \frac{x+\sin(2x)}{x} = \lim\limits_{x\to 0} \left(\frac{x}{x} + \frac{\sin(2x)}{x}\right)$

$= \lim\limits_{x\to 0} \left(1 + \frac{2\sin(2x)}{2x}\right)$

$= \lim\limits_{x\to 0} (1) + 2 \cdot \lim\limits_{x\to 0} \left(\frac{\sin(2x)}{2x}\right)$

$= 1 + 2 \cdot 1$

$= 3$

d) $\lim\limits_{x\to -5} \frac{x^2+6x+5}{x^2+2x-15}$

$= \lim\limits_{x\to -5} \frac{(x+5)(x+1)}{(x+5)(x-3)}$

$= \lim\limits_{x\to -5} \frac{x+1}{x-3} = \frac{-5+1}{-5-3} = \frac{-4}{-8} = \frac{1}{2}.$

4. (11 points) Suppose $f(x) = \frac{3}{x}$

a) Show that $f'(x) = -\frac{3}{x^2}$ using the limit definition of derivative. No credit will be awarded for any other method.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{3}{x+h} - \frac{3}{x}}{h} = \lim_{h \to 0} \frac{3x - 3(x+h)}{x(x+h)} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{-3h}{x(x+h) \cdot h}$$

$$= \lim_{h \to 0} \frac{-3}{x(x+h)}$$

$$= \frac{-3}{x(x+0)}$$

$$= \frac{-3}{x^2} \checkmark$$

b) Find the equation of the line tangent to f at the point where $x = 6$. Any form of linear equation is acceptable.

$$f(6) = \frac{3}{6} = \frac{1}{2}$$

$$f'(6) = -\frac{3}{(6)^2} = -\frac{3}{36} = -\frac{1}{12}$$

$$y - \frac{1}{2} = -\frac{1}{12}(x-6)$$

5. (15 points) Find the indicated derivatives for each expression. You may use differentiation shortcuts. You do not need to simplify your answers.

a) $\frac{d}{dx}[(2x^2-3)^3(5-2x)^4]$

$= (2x^2-3)^3 4(5-2x)^3 \cdot (-2) + (5-2x)^4 \cdot 3(2x^2-3)^2 \cdot (4x)$

b) $\frac{d}{dx}\left[\frac{\sin^2 x}{\cos(x)+1}\right] = \frac{(\cos(x)+1) \cdot 2\sin(x)\cos(x) - \sin^2(x) \cdot (-\sin(x))}{(\cos(x)+1)^2}$

c) $\frac{d}{dx}\left[\tan^3\left(\frac{2}{x}\right)\right] = 3\tan^2\left(\frac{2}{x}\right) \cdot \sec^2\left(\frac{2}{x}\right) \cdot -\frac{2}{x^2}$

6. (10 points) Suppose $s(t) = 2t^2 + 3$ gives the position, in inches, of an object after t seconds, for all $t \geq 0$.

a) Find the average velocity of the object over the interval from $t = 1$ to $t = 7$. Express your answer with appropriate units.

$$V_{avg} = \frac{s(7) - s(1)}{7-1} = \frac{(2(7)^2+3)-(2(1)^2+3)}{6}$$

$$= \boxed{16 \text{ in/s}}$$

b) Find the instantaneous velocity of the object at $t = 3$.

$$s'(t) = v(t) = 4t$$

$$v(3) = \boxed{12 \text{ in/s}}$$

7. (12 points) Suppose $f(x) = \begin{cases} \frac{1}{x} & \text{if } x < -1 \\ \cos(\pi x) & \text{if } -1 \leq x < 2 \\ \sqrt{x-1} & \text{if } 2 < x \leq 5 \\ x^2 - 4x & \text{if } x > 5 \end{cases}$

a) Is f continuous at $x = -1$? Justify your answer completely.

b) Is f continuous at $x = 2$? Justify your answer completely.

c) Is f continuous at $x = 5$? Justify your answer completely.

7. (12 points) Suppose f is a piecewise-defined function given by the equation:

$$f(x) = \begin{cases} \frac{1}{x} & \text{if } x < -1 \\ \cos(\pi x) & \text{if } -1 \le x < 2 \\ \sqrt{x-1} & \text{if } 2 < x \le 5 \\ x^2 - 4x & \text{if } x > 5 \end{cases}$$

a) Is f continuous at $x = -1$? Justify your answer completely.

$f(-1) = \cos(-\pi) = -1$

$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} \frac{1}{x} = \frac{1}{-1} = -1$

$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} \cos(\pi x) = \cos(-\pi) = -1$

So $\lim_{x \to -1} f(x) = f(-1)$. f is continuous at $x = -1$.

b) Is f continuous at $x = 2$? Justify your answer completely.

$f(2)$ is undefined. Hence f is not continuous at $x = 2$.

c) Is f continuous at $x = 5$? Justify your answer completely.

$\lim_{x \to 5^-} f(x) = \lim_{x \to 5^-} \sqrt{x-1} = \sqrt{5-1} = \sqrt{4} = 2$

$\lim_{x \to 5^+} f(x) = \lim_{x \to 5^+} x^2 - 4x = 5^2 - 4(5) = 25 - 20 = 5$

Hence $\lim_{x \to 5} f(x)$ DNE.

Thus f is not continuous at $x = 5$.

8. (12 points) Suppose f and g are both differentiable functions for their entire domains. Function values and values of their derivatives when $x = 5$ and $x = -3$ are given in the following table:

x	f	g	f'	g'
5	2	-3	4	6
-3	8	0	-2	1

Using the above information, determine the following values, if they exist. If a value does not exist, explain why. Show your work.

a) $(f+g)'(-3)$ $= f'(-3) + g'(-3)$

$= -2 + 1$

$= -1$

b) $(fg)'(5)$ $= f(5) \cdot g'(5) + g(5) \cdot f'(5)$

$= 2 \cdot 6 + (-3) \cdot 4$

$= 12 - 12 = 0$

c) $\left(\frac{f}{g}\right)'(-3)$ Note that $g(-3) = 0$.

Thus $\left(\frac{f}{g}\right)'(-3)$ does not exist.

d) $(f \circ g)'(5)$ $= f'(g(5)) \cdot g'(5)$

$= f'(-3) \cdot g'(5)$

$= -2 \cdot 6$

$= -12$