1.[10]	2.[20]	3.[10]	4.[5]	5.[15]	6.[10]	7.[5]	8.[15]	9.[10]	Total: [100]

Math 225 Midterm

February 13, 2017

Name: Solutions

Section:

Instructions: Closed book and closed notes. Answers must include supporting work. Calculators and cell phones out of sight.

1.(10pts) Give an answer of True or False for the following.

T

(a) For any continuous function f on the interval $[a, b]$ we have that

$$\frac{d}{dx}\left(\int_a^b f(x)dx\right) = 0.$$

F

(b) If f and g are continuous on $[a, b]$, then

$$\int_a^b f(x)g(x)dx = \int_a^b f(x)dx \int_a^b g(x)dx.$$

F

(c) If $\int_0^1 f(x)dx = 0$, then $f(x) = 0$ for all x in interval $[0, 1]$.

T

(d) If f and g are continuous and $f(x) \leq g(x)$ on $[a, b]$, then

$$\int_a^b f(x)dx \leq \int_a^b g(x)dx.$$

T

(e) It is sometimes the case that

$$\int_0^x f'(t)dt = f(x).$$

2.(20pts) Evaluate the following integrals.

(a) $\int (1 + \tan t)^3 \sec^2 t \, dt$ $= \int u^3 du = \frac{u^4}{4} + C$

$u = 1 + \tan t$

$du = \sec^2 t \, dt$

$= \boxed{\frac{(1+\tan t)^4}{4} + C}$

(b) $\int_0^{\pi/6} \frac{\sin x}{\cos^3 x} dx$ $= -\int_1^{\sqrt{3}/2} u^{-3} du$

$u = \cos x$

$du = -\sin x \, dx$

$x = 0 \Rightarrow u = \cos(0) = 1$

$x = \pi/6 \Rightarrow u = \cos(\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$

$= \frac{1}{2u^2} \Big|_1^{\sqrt{3}/2}$

$= \frac{2}{3} - \frac{1}{2} = \boxed{1/6}$

(c) $\int s^3 \sqrt{s^2 + 1} ds$ $= \frac{1}{2} \int s^2 \sqrt{u} \, du$

$u = s^2 + 1$

$du = 2s \, ds$

$\downarrow$

$s^2 = u - 1$

$= 1/2 \int (u-1) \sqrt{u} \, du$

$= 1/2 \int u^{3/2} - u^{1/2} du$

$= \boxed{1/5 (s^2+1)^{5/2} - 1/3 (s^2+1)^{3/2} + C}$

3.(10pts) Determine the following.

(a) $\int_5^7 f(x)dx$, given that $\int_2^7 f(x)dx = 8$ and $\int_2^5 f(x)dx = 3$.

$$\int_5^7 f(x)dx = \int_2^7 f(x)dx - \int_2^5 f(x)dx$$

$$= 8 - 3 = \boxed{5}$$

(b) $\int_2^8 f(3x)dx$, given that $\int_6^{24} f(x)dx = 9$.

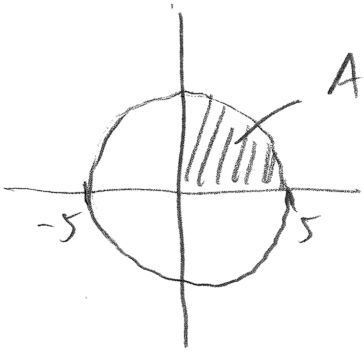
$$\int_2^8 f(3x)dx = \frac{1}{3}\int_6^{24} f(u)du = \frac{1}{3}(9)$$

$$2\text{ pts}\left\{\begin{array}{l} u = 3x \\ du = 3dx \end{array}\right. \quad \left| \begin{array}{l} x = 2 \Rightarrow u = 6 \\ x = 8 \Rightarrow u = 24 \end{array}\right. \qquad = \boxed{3}$$

4.(5pts) Evaluate the integral by interpreting it in terms of areas.

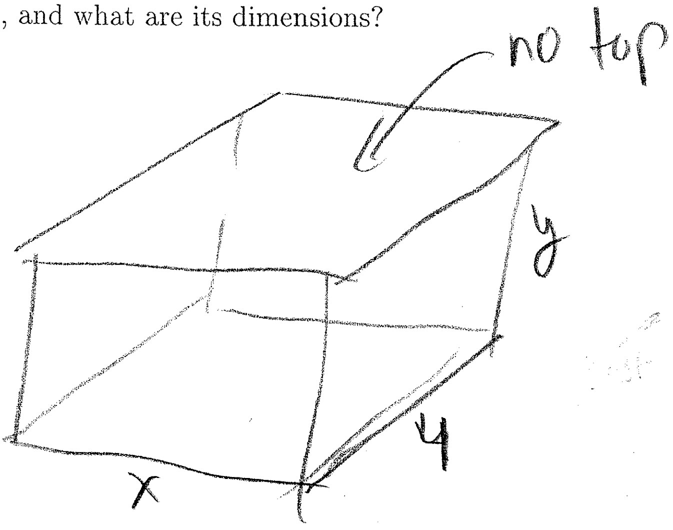
$$\int_0^5 \sqrt{25 - x^2}dx.$$

$$= \boxed{\frac{25\pi}{4}}$$



$$\text{Area} = \frac{\pi(5)^2}{4}$$

5.(15pts) A tank with rectangular base base and rectangular sides is open at the top. It is to be constructed so that its width is 4 meters and its volume is 36 cubic meters. If building the tank costs $10/m^2$ for the base and $5/m^2$ for the sides, what is the cost of the least expensive tank, and what are its dimensions?



minimize (cost)

$C = 10(4x) + 5(2xy) + 5(8y)$

$= 40x + 10xy + 40y$

Constraint (volume)

$4xy = 36$

$\hookrightarrow y = 9/x$

$C(x) = 40x + 90 + \frac{360}{x}$

$C'(x) = 40 - \frac{360}{x^2} = \frac{40(x^2-9)}{x^2}$

Critical points: $x = 3, \cancel{x=-3}, \cancel{x=0}$

Check $x=3$ minimum:

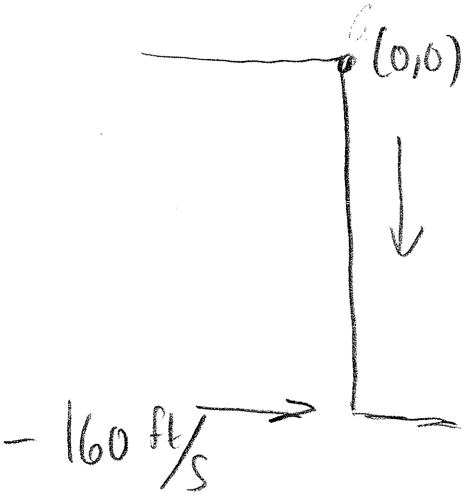
2nd derivative test:

$C''(x) = \frac{360(2)}{x^3}$

$C''(3) > 0$, so min

$$\boxed{\begin{array}{l}\text{Dimensions: } 3 \times 4 \times 3 \\ \text{Cost: } \$330\end{array}}$$

6.(10pts) A penny is dropped from the top of a building and hits the ground with a speed of $160 ft/s$. Determine the height of the building. (You can assume acceleration due to gravity is $32 ft/s^2$)



$a(t) = -32$

$v(t) = -32t + C \qquad v(0) = 0$

$\Rightarrow C = 0$

$= -32t$

$s(t) = -16t^2 + D \qquad s(0) = 6$

$\Rightarrow D = 0$

$v(t) = -32t = -160$

$\Rightarrow t = 5$

$\text{height} = |s(5)| = \boxed{400 ft}$

7.(5pts) Compute the following derivative.

$$\frac{d}{dx}\int_{-\sqrt{x^2+1}}^{x^3} t^2\tan(t)dt.$$

$= (x^3)^2 \tan(x^3) \cdot 3x^2$

$+ (-\sqrt{x^2+1})^2 \tan(-\sqrt{x^2+1})\left(-\frac{x}{\sqrt{x^2+1}}\right)$

8.(15pts) The velocity function (in meters per second) for an object moving in a straight line is $v(t) = 6t^2 - 18t + 12$. Find the following for time $t = 0$ to $t = 3$.

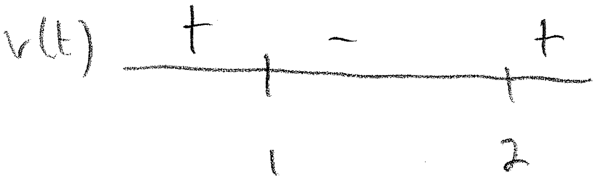
(a) The displacement.

(b) The total distance traveled.

a) $\int_0^3 6t^2-18t+12\,dt = 2t^3-9t^2+12t\Big|_0^3$

$= \boxed{9}$

b) $v(t) = 6(t-2)(t-1)$



$\int_0^3 |v(t)|\,dt = \int_0^1 6t^2-18t+12\,dt - \int_1^2 6t^2-18t+12\,d$

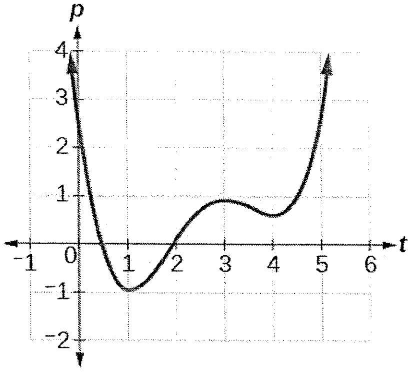
$+ \int_2^3 6t^2-18t+12\,dt$

In particular,

$\int_0^3 |v(t)|\,dt = \int_0^3 6t^2-18t+12\,dt - 2\int_1^2 6t^2-18t+12\,d$

$= 9 + 2 = \boxed{11}$

9.(10pts) Below is the graph of a function p defined on the entire real line $(-\infty, \infty)$.



Define

$$h(x) = \int_0^x p(t)dt.$$

(a) On what intervals is h increasing/decreasing? Explain.

$h'(x) = p(x)$ $\quad$ $p(x) > 0$ on $(-\infty, \tfrac{1}{2}) \cup (2, \infty)$

$p(x) < 0$ on $(-\tfrac{1}{2}, 2)$

$$\boxed{\begin{array}{l}\text{Increasing: } (-\infty, \tfrac{1}{2}) \cup (2, 8) \\ \text{decreasing: } (-\tfrac{1}{2}, 2)\end{array}}$$

(b) On what intervals is h concave up/concave down? Explain.

$h''(x) = p'(x)$

$$\boxed{\begin{array}{l}\text{Concave up: } (1,3) \cup (4, \infty) \\ \text{Concave down: } (-\infty, 1) \cup (3,4)\end{array}}$$