Math 225 Final Examination Fall 2015

Name (print) KEY

Name (sign)____________________

Bing ID number ____________________
(Your ID may be checked during or after the test)

Section number_____ Instructor____________________

No books, no notes, no electronic devices (calculators, cell phones, smart watches, etc.) Write all your work on the test – nothing else will be graded. **You must show all your work.** Your work must be legible, and the final answers must be reasonably simplified.

On some problems you may be asked to use a specific method to solve the problem (for instance, "Use the Fundamental Theorem of Calculus to find..."). On all other problems, you may use any method we have covered. **You may not use methods that we have not covered.**

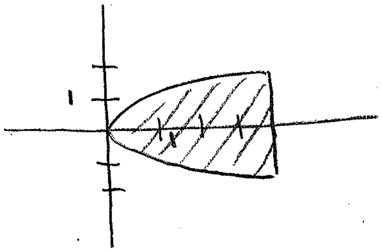
Wandering Eyes Policy

You must keep your eyes on your own work at all times. If you are found looking around, you will be warned once, and only once. A second infraction may result in automatic zero on this test, and possibly a referral to the Harpur College Academic Honesty Committee.

For instructor's use only:

1. ____ (10)	4. ____ (10)	7. ____ (15)
2. ____ (15)	5. ____ (10)	8. ____ (15)
3. ____ (15)	6. ____ (10)	**TOTAL:**

1. (10 points) Find the area of the region bounded by the curves $x = y^2$ and $x = 4$.



$$y^2 = 4$$
$$y = \pm 2$$

$$A = \int_{-2}^{2} (4 - y^2)\,dy$$
$$= 4y - \frac{y^3}{3} \Big|_{-2}^{2}$$
$$= \left(8 - 8/3\right) - \left(-8 + 8/3\right)$$
$$= \boxed{32/3}$$

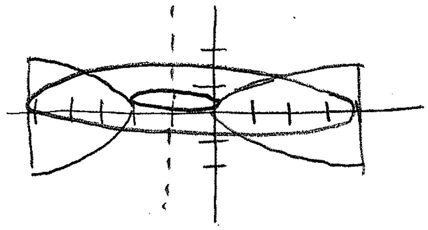
or

$$y^2 = x$$
$$y = \sqrt{x}$$

$$A = \int_0^4 2\sqrt{x}\,dx = 2\,{}^2\!/_3\, x^{3/2} \Big|_0^4 = {}^4\!/_3\, 4^{3/2} = \boxed{\frac{32}{3}}$$

2. (15 points) The region bounded by the curves $x = y^2$ and $x = 4$ is rotated about the line $x = -1$.

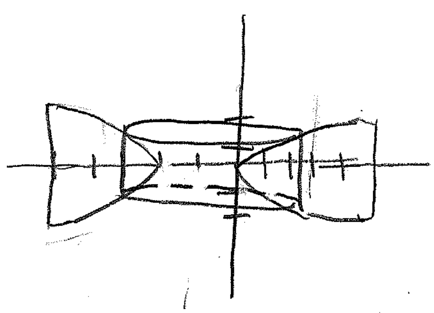
a) Set up the integral that represents the volume of the resulting solid using the **washers** method. You must draw a typical washer or otherwise justify your formula. **Do not evaluate the integral.**



$$\int_{-2}^{2} \pi(4-(-1))^2 - \pi(y^2-(-1))^2\,dy$$

$$= \pi\int_{-2}^{2} 25-(y^2+1)^2\,dy$$

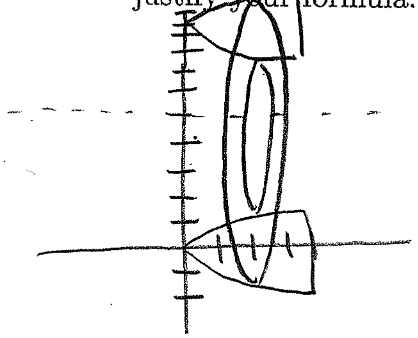
b) Set up the integral that represents the volume of the resulting solid using the **shells** method. You must draw a typical shell or otherwise justify your formula. **Do not evaluate the integral.**



$$\int_{0}^{4} 2\pi(x+1)(2\sqrt{x})\,dx$$

3. (15 points) The region bounded by the curves $x = y^2$ and $x = 4$ is rotated about the line $y = 5$.

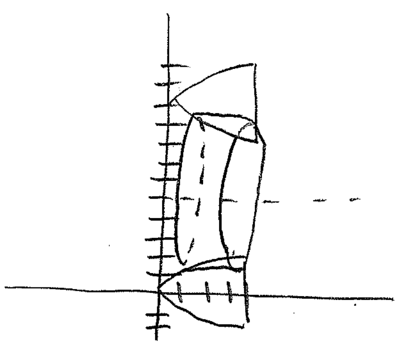
a) Set up the integral that represents the volume of the resulting solid using the **washers** method. You must draw a typical washer or otherwise justify your formula. **Do not evaluate the integral.**



$$\int_0^4 \pi(5-(-\sqrt{x}))^2 - \pi(5-\sqrt{x})^2\,dx$$

$$= \pi \int_0^4 \left((5+\sqrt{x})^2 - (5-\sqrt{x})^2\right)dx$$

b) Set up the integral that represents the volume of the resulting solid using the **shells** method. You must draw a typical shell or otherwise justify your formula. **Do not evaluate the integral.**



$$\int_{-2}^{2} 2\pi(5-y)(4-y^2)\,dy$$

4. (10 points) Evaluate the integral (definite or indefinite).

a) $\int_0^{\pi} x \cdot \sin(4x)\, dx$

$u = x \quad dv = \sin(4x)dx$

$du = dx \quad v = -1/4 \cos(4x)$

$$= -x/4 \cos(4x) \Big|_0^{\pi} - \int_0^{\pi} -1/4 \cos(4x)\, dx$$

$$= -\pi/4 \cos(4\pi) + 1/16 \sin(4x) \Big|_0^{\pi}$$

$$= -\pi/4 + 1/16(\sin(4\pi) - \sin(0))$$

$$= -\pi/4 + 0$$

$$= \boxed{-\pi/4}$$

b) $\int \sqrt{x} \cdot \ln x\, dx$

$u = \ln x \quad dv = x^{1/2} dx$

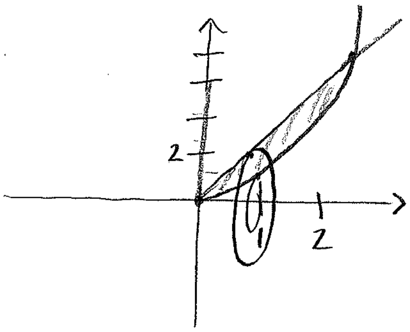
$du = \frac{1}{x} dx \quad v = 2/3\, x^{3/2}$

$$= 2/3\, x^{3/2} \ln x - \int 2/3\, x^{3/2} (1/x) dx$$

$$= 2/3\, x^{3/2} \ln x - 2/3 \int x^{1/2} dx$$

$$= \boxed{2/3\, x^{3/2} \ln x - 4/9\, x^{3/2} + C}$$

5. (10 points) Find the volume of the solid obtained by rotating the region bounded by $y = x^3$, $y = 4x$, $x \geq 0$ about the x-axis.



$4x = x^3$

$x^3 - 4x = 0$

$x(x^2 - 4) = 0$

$x = 0, \pm 2$

$$V = \int_0^2 \pi\left((4x)^2 - \pi(x^3)^2\right)dx$$

$$= \pi \int_0^2 (16x^2 - x^6)\,dx$$

$$= \pi\left(16\frac{x^3}{3} - \frac{x^7}{6}\right)\Big|_0^2$$

$$= \pi\left(\frac{16 \cdot 8}{3} - \frac{2^7}{6}\right)$$

$$= \boxed{\frac{512\pi}{21}}$$

6. (10 points) A catapult shoots a ball from the ground level straight up in the air with the initial velocity of 20 m/s. Find when the ball is 15 m above the ground on the way down. (The time is measured in seconds, starting from the time the ball is shot).

NOTE: Assume that the acceleration of gravity is 10 m/s^2 and ignore the drag.

$V(0) = 20$

$S(0) = 0$

$S(t) = 15$

find

$a(t) = -10$

$V(t) = -10t + C$

$V(0) = C = 20$

$V(t) = -10t + 20$

$S(t) = -5t^2 + 20t + K$

$S(0) = K = 0$

$S(t) = -5t^2 + 20t = 15$

$-5t^2 + 20t - 15 = 0$

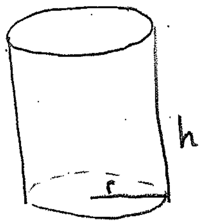
$t^2 - 4t + 3 = 0$

$(t-3)(t-1) = 0$

$t = 3, 1$

Way down: $\boxed{t = 3 \text{ sec}}$

7. (15 points) What is the smallest possible total surface area (top, bottom and side) of the right circular cylinder of volume (16π) m^3? Justify.



$$V = \pi r^2 h = 16\pi$$

$$h = \frac{16}{r^2}$$

$$SA = 2\pi r^2 + 2\pi r h$$

$$= 2\pi r^2 + 2\pi r\left(\frac{16}{r^2}\right)$$

$$= 2\pi r^2 + \frac{32\pi}{r}$$

$$SA' = 4\pi r - \frac{32\pi}{r^2}$$

SA' DNE at $r=0$, but this is not in the domain, so ignore it.

$$\left(4\pi r - \frac{32\pi}{r^2} = 0\right) r^2$$

$$4\pi r^3 - 32\pi = 0$$

$$r^3 = 8$$

$$r = 2$$

$$SA'' = 4\pi + \frac{64\pi}{r^3} \geq 0$$

$\Rightarrow r = 2$ is a min!

$$SA = 2\pi(2^2) + \frac{32\pi}{2}$$

$$= 8\pi + 16\pi$$

$$= \boxed{24\pi \text{ m}^2}$$

8. (15 points)

a) Evaluate the integral

$$\int_0^{\pi} x^2 \sin x \, dx$$

$u = x^2 \quad dv = \sin x \, dx$

$du = 2x\,dx \quad v = -\cos x$

$= -x^2 \cos x \Big|_0^{\pi} + \int_0^{\pi} 2x \cos x \, dx$

$\bar{u} = 2x \qquad d\bar{v} = \cos x \, dx$

$d\bar{u} = 2\,dx \quad \bar{v} = \sin x \, dx$

$= -x^2 \cos x \Big|_0^{\pi} + \left[2x \sin x \Big|_0^{\pi} - \int_0^{\pi} 2 \sin x \, dx \right]$

$= -x^2 \cos x \Big|_0^{\pi} + 2x \sin x \Big|_0^{\pi} + 2 \cos x \Big|_0^{\pi}$

$= -\pi^2 \cos(\pi) + 0 + 2\pi \sin \pi - 0 + 2 \cos \pi - 0$

$= \pi^2 + 0 - 2 - 2$

$= \boxed{\pi^2 - 4}$

b) Find the average value on the segment $[0, \pi]$ of the function

$$f(x) = x^2 \sin x$$

$f_{avg} = \frac{1}{\pi} \int_0^{\pi} x^2 \sin x \, dx$

$= \frac{1}{\pi} (\pi^2 - 4)$

$= \boxed{\pi - 4/\pi}$