Math 225 Final Examination Fall 2015

Name (print) KEY

Name (sign) ____________________________

Bing ID number ____________________________
(Your ID may be checked during or after the test)

Section number ______  Instructor ____________________________

No books, no notes, no electronic devices (calculators, cell phones, smart watches, etc.) Write all your work on the test – nothing else will be graded. You must show all your work. Your work must be legible, and the final answers must be reasonably simplified.

On some problems you may be asked to use a specific method to solve the problem (for instance, “Use the Fundamental Theorem of Calculus to find...”). On all other problems, you may use any method we have covered. You may not use methods that we have not covered.

Wandering Eyes Policy

You must keep your eyes on your own work at all times. If you are found looking around, you will be warned once, and only once. A second infraction may result in automatic zero on this test, and possibly a referral to the Harpur College Academic Honesty Committee.

For instructor’s use only:

1. ___ (10)  4. ___ (10)  7. ___ (15)
2. ___ (15)  5. ___ (10)  8. ___ (15)
3. ___ (15)  6. ___ (10)  TOTAL: __________

1
1. (10 points) Find the area of the region bounded by the curves \( x = y^2 \) and \( x = 4 \).

\[
\begin{align*}
\text{or} & \\
A = & \quad \int_{0}^{4} 2\sqrt{x} \, dx \\
& = 2^{2/3} x^{3/2} \bigg|_{0}^{4} \\
& = \frac{4}{3} \cdot 4^{3/2} = \frac{32}{3}
\end{align*}
\]
2. (15 points) The region bounded by the curves \( x = y^2 \) and \( x = 4 \) is rotated about the line \( x = -1 \).

   a) Set up the integral that represents the volume of the resulting solid using the \textbf{washers} method. You must draw a typical washer or otherwise justify your formula. \textbf{Do not evaluate the integral.}

   \[
   \int_{-2}^{2} \pi (4-(c-1))^2 - \pi (y^2-(c-1))^2 \, dy
   \]

   \[
   = \pi \int_{-2}^{2} 25 - (y^2+1)^2 \, dy
   \]

   b) Set up the integral that represents the volume of the resulting solid using the \textbf{shells} method. You must draw a typical shell or otherwise justify your formula. \textbf{Do not evaluate the integral.}

   \[
   \int_{0}^{4} 2\pi (x+1)(2\sqrt{x}) \, dx
   \]
3. (15 points) The region bounded by the curves $x = y^2$ and $x = 4$ is rotated about the line $y = 5$.
   a) Set up the integral that represents the volume of the resulting solid using the washers method. You must draw a typical washer or otherwise justify your formula. Do not evaluate the integral.

   \[
   \int_0^4 \pi \left(5 - (-\sqrt{x})\right)^2 - \pi \left(5 - \sqrt{x}\right)^2 \, dx
   \]

   \[
   = \pi \int_0^4 \left((5 + \sqrt{x})^2 - (5 - \sqrt{x})^2\right) \, dx
   \]

   b) Set up the integral that represents the volume of the resulting solid using the shells method. You must draw a typical shell or otherwise justify your formula. Do not evaluate the integral.

   \[
   \int_{-2}^{2} 2\pi (5-y)(4-y^2) \, dy
   \]
4. (10 points) Evaluate the integral (definite or indefinite).

\( a) \int_0^\pi x \cdot \sin(4x) \, dx \)

\[ u = x \quad dv = \sin(4x) \, dx \]

\[ du = dx \quad v = \frac{-1}{4} \cos(4x) \]

\[ = \left. -\frac{x}{4} \cos(4x) \right|_0^\pi - \int_0^\pi -\frac{1}{4} \cos(4x) \, dx \]

\[ = -\frac{\pi}{4} + \frac{1}{16} \sin(4\pi) \int_0^\pi \]

\[ = -\frac{\pi}{4} + \frac{1}{16} (\sin(4\pi) - \sin(0)) \]

\[ = -\frac{\pi}{4} + 0 \]

\[ = \left[ -\frac{\pi}{4} \right] \]

\( b) \int \sqrt{x} \cdot \ln x \, dx \)

\[ u = \ln x \quad dv = \sqrt{x} \, dx \]

\[ du = \frac{1}{x} \, dx \quad v = \frac{2}{3} x^{3/2} \]

\[ = \frac{2}{3} x^{3/2} \ln x - \int \frac{2}{3} x^{3/2} \left( \frac{1}{x} \right) \, dx \]

\[ = \frac{2}{3} x^{3/2} \ln x - \frac{2}{3} \int x^{1/2} \, dx \]

\[ = \frac{2}{3} x^{3/2} \ln x - \frac{4}{9} x^{3/2} + C \]
5. (10 points) Find the volume of the solid obtained by rotating the region bounded by \( y = x^3 \), \( y = 4x \), \( x \geq 0 \) about the \( x \)-axis.

\[ 4x = x^3 \]
\[ x^3 - 4x = 0 \]
\[ x(x^2 - 4) = 0 \]
\[ x = 0, 2 \]

\[ V = \int_0^2 \pi \left( (4x)^2 - (x^3)^2 \right) dx \]

\[ = \pi \int_0^2 (16x^2 - x^6) \, dx \]

\[ = \pi \left[ 16\frac{x^3}{3} - \frac{x^7}{7} \right]_0^2 \]

\[ = \pi \left( \frac{128}{3} - \frac{128}{7} \right) \]

\[ = \frac{512\pi}{21} \]
6. (10 points) A catapult shoots a ball from the ground level straight up in the air with the initial velocity of 20 m/s. Find when the ball is 15 m above the ground on the way down. (The time is measured in seconds, starting from the time the ball is shot).

NOTE: Assume that the acceleration of gravity is 10 m/s² and ignore the drag.

\[ v(0) = 20 \quad a(t) = -10 \]
\[ s(0) = 0 \quad v(t) = -10t + c \]
\[ s(t) = 15 \quad v(0) = c = 20 \]
\[ v(t) = -10t + 20 \]

\[ s(t) = -5t^2 + 20t + k \]
\[ s(0) = k = 0 \]
\[ s(t) = -5t^2 + 20t = 15 \]

\[ -5t^2 + 20t = 15 = 0 \]
\[ t^2 - 4t + 3 = 0 \]
\[ (t-3)(t-1) = 0 \]
\[ t = 3, 1 \]

Way down: \[ t = 3 \text{ sec} \]
7. (15 points) What is the smallest possible total surface area (top, bottom and side) of the right circular cylinder of volume \((16\pi)\) m\(^3\)? Justify.

\[ V = \pi r^2 h = 16\pi \]

\[ h = \frac{16}{r^2} \]

\[ SA = 2\pi r^2 + 2\pi rh \]
\[ = 2\pi r^2 + 2\pi r \left( \frac{16}{r^2} \right) \]
\[ = 2\pi r^2 + \frac{32\pi}{r} \]

\[ SA' = 4\pi r - \frac{32\pi}{r^2} \]

SA' DNE at \(r = 0\), but this is not in the domain, so ignore it.

\[ (4\pi r - \frac{32\pi}{r^2} = 0)r^2 \]

\[ 4\pi r^3 - 32\pi = 0 \]

\[ r^3 = 8 \]

\[ r = 2 \]

\[ SA'' = 4\pi + \frac{64\pi}{r^3} \geq 0 \]

\[ \Rightarrow r = 2 \text{ is a min!} \]

\[ SA = 2\pi (2^2) + \frac{32\pi}{2} \]

\[ = 8\pi + 16\pi \]

\[ = 24\pi \text{ m}^2 \]
8. (15 points)
   a) Evaluate the integral

   \[ \int_0^\pi x^2 \sin x \, dx \]

   \[ = x^2 \cos x \bigg|_0^\pi - \int_0^\pi 2x \cos x \, dx \]

   \[ = -x^2 \cos x \bigg|_0^\pi + 2x \sin x \bigg|_0^\pi \]

   \[ = -\pi^2 \cos(\pi) + 0 + 2\pi \sin \pi = 0 + 2 \cos \pi - 0 \]

   \[ = -\pi^2 + 0 - 2 - 2 \]

   \[ = \boxed{\pi^2 - 4} \]

   b) Find the average value on the segment \([0, \pi]\) of the function

   \[ f(x) = x^2 \sin x \]

   \[ f_{\text{avg}} = \frac{1}{\pi} \int_0^\pi x^2 \sin x \, dx \]

   \[ = \frac{1}{\pi} \left( \pi^2 - 4 \right) \]

   \[ = \boxed{\pi^2 - 4/\pi} \]