1.[10]	2.[20]	3.[10]	4.[5]	5.[15]	6.[15]	7.[10]	8.[5]	9.[10]	Total: [100]

Math 224 Midterm

February 08, 2017

Name: Solutions Section:

Instructions: Closed book and closed notes. Answers must include supporting work. Calculators and cell phones out of sight.

1.(10pts) Give an answer of True or False for the following.

F **(a)** It is always the case that for any functions $f(x)$ and $g(x)$ we have that

$$\lim_{x\to a} f(x)g(x) = \lim_{x\to a} f(x) \lim_{x\to a} g(x).$$

T **(b)** If f is differentiable at the real number a then f is continuous at a.

F **(c)** If the function f has a limit at a, then f is continuous at a.

F **(d)** The derivative of $\frac{f(x)}{g(x)}$ is $\frac{f'(x)}{g'(x)}$.

F **(e)** If $f(x)$ approaches 0 when $x \to 0$, then we must also have that $f'(x)$ approaches 0 as $x \to 0$.

5 pts each

2.(20pts) Compute the following limits. If the limit does not exist, then explain why.

(a) $\lim_{x \to 2} \frac{4x-8}{|x^2-4|}$

[redacted] $\lim_{x \to 2^-} \frac{4x-8}{-(x^2-4)} = \lim_{x \to 2^-} \frac{4\cancel{(x-2)}}{-\cancel{(x-2)}(x+2)} = -1$

[redacted] $\lim_{x \to 2^+} \frac{4(x-2)}{x^2+4} = 1$

$\boxed{1 \neq -1, \text{ so DNE}}$

[redacted]

(b) $\lim_{s \to 5^+} \frac{s^2-25}{2s^2-20s+50}$

[redacted] $\lim_{s \to 5^+} \frac{(s+5)\cancel{(s-5)}}{2(s-5)^{\cancel{2}}} = \lim_{s \to 5^+} \frac{s+5}{2(s-5)} = \boxed{\infty}$

[redacted]

(c) $\lim_{x \to 0} \frac{\sin(3x^2)}{(x-1)\sin(2x^2)} = \lim_{x \to 0} \frac{\sin(3x^2)}{\sin(2x^2)} \cdot \lim_{x \to 0} \frac{1}{x-1}$ [redacted]

$= \left(\frac{3}{2}\right)\left(-1\right) = \boxed{-\frac{3}{2}}$

[redacted] [redacted]

(d) $\lim_{u \to 2} \frac{\sqrt{4u+1}-3}{u-2}$

$= \lim_{u \to 2} \frac{\sqrt{4u+1}-3}{u-2} \cdot \frac{\sqrt{4u+1}+3}{\sqrt{4u+1}+3}$ [redacted]

$= \lim_{u \to 2} \frac{4u+1-9}{(u-2)(\sqrt{4u+1}+3)} = \lim_{u \to 2} \frac{4\cancel{(u-2)}}{\cancel{(u-2)}(\sqrt{4u+1}+3)}$ [redacted]

[redacted] $= \boxed{2/3}$

3.(10pts) Find the equation of the tangent line to the graph of g at the point $(1, 1)$.

$$g(x) = (6x - 5)^6.$$

You may leave your answer in point-slope form. [redacted]

$$g'(x) = 6\left((6x-5)^5\right)\cdot(6)$$ [redacted]

$$g'(1) = 36$$ [redacted]

$$y - 1 = 36(x-1)$$ [redacted]

4.(5pts) Use the Intermediate Value Theorem to show that there is a number c in the given interval such that $f(c) = 0$.

$$f(x) = x^2 - x - \sin x, \quad (1, 2).$$

[redacted] $f(1) = -\sin(1) < 0$

[redacted] $f(2) = 2 - \sin(2) > 0$ as $|\sin x| \leq 1$

[redacted] By IVT, there exists c in $(1,2)$ such that $f(c) = 0$.

5 pts each

5.(15pts) Find the derivatives of the following functions. You do not need to simplify your answer.

(a) $f(t) = \tan\left(\frac{t}{t^3+1}\right)$

$$f'(t) = \underbrace{\sec^2\left(\frac{t}{t^3+1}\right)}\underbrace{\left[\frac{(1)(t^3+1) - t(3t^2)}{(t^3+1)^2}\right]}$$

[redacted] [redacted]

(b) $h(x) = \cos(x^3 \sin x)$

$$h'(x) = \underbrace{-\sin(x^3 \sin x)}\underbrace{(3x^2 \sin x + x^3 \cos x)}$$

[redacted] [redacted]

(c) $g(y) = (4y-1)^4(6-3y)^{-3}$

[redacted]

$$g'(y) = 4(4y-1)^3(4)(6-3y)^{-3}$$

[redacted]

$$+ (4y-1)^4 \underbrace{(-3)(6-3y)^{-4}}\underbrace{(-3)}$$

[redacted] [redacted]

6.(15pts) You are given the following piecewise defined function.

$$f(x) = \begin{cases} x^2 - 4x + 8, & x \leq 3; \\ 2x - 1, & 3 < x < 4; \\ \sqrt{x^3 - 28}, & x \geq 4. \end{cases}$$

(a) Where is $f(x)$ continuous? Justify your answer. $(-\infty, 4) \cup (4, \infty)$

(b) Where is $f(x)$ differentiable? Justify your answer. $(-\infty, 4) \cup (4, \infty)$

(c) Write down a formula for $f'(x)$.

a) $\left.\begin{array}{l} \lim_{x \to 3^-} f(x) = 5 \\ \lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} 2x-1 = 5 \end{array}\right\}$ f is continuous at $x = 3$

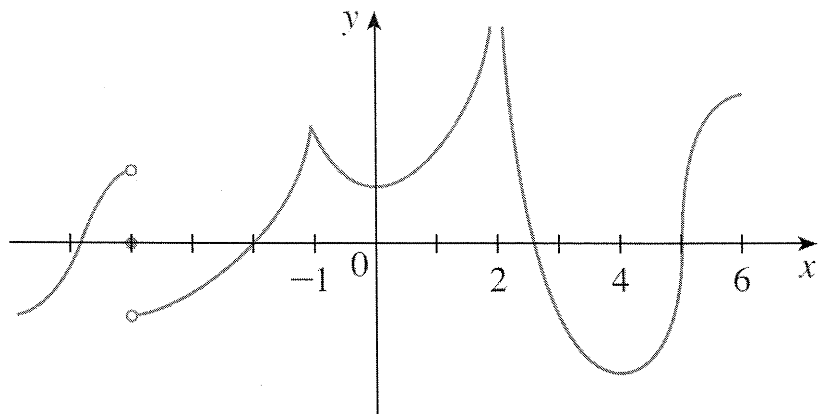
$\left.\begin{array}{l} \lim_{x \to 4^-} f(x) = \lim_{x \to 4^-} 2x-1 = 7 \\ \lim_{x \to 4^+} f(x) = \lim_{x \to 4^+} \sqrt{x^3-28} = 6 \end{array}\right\}$ f is not continuous at $x = 4$

b) f is not continuous at $x=4$, so f is not differentiable at $x=4$

$\left.\begin{array}{l} \lim_{x \to 3^-} f'(x) = \lim_{x \to 3^-} 2x-4 = 2 \\ \lim_{x \to 3^+} f'(x) = \lim_{x \to 3^+} 2 = 2 \end{array}\right\}$ f is differentiable at $x = 3$

c)
$$f'(x) = \begin{cases} 2x-4 & x \leq 3 \\ 2 & 3 < x < 4 \\ \dfrac{3x^2}{2\sqrt{x^3-28}} & x > 4 \end{cases}$$

7.(10pts) The graph of f is shown.



(a) State the numbers at which f is not continuous. Briefly explain your answer.

(b) State the numbers at which f is not differentiable. Briefly explain your answer.

a) $x = -4$ jump discontinuity

$x = 2$ vertical asymptote

b) $x = 2, -4$ since f is not continuous at these values

$x = -1$ graph has a kink.

8.(5pts) The position function of a particle is given by $s(t) = 3t^2 - t^3$ where $t \geq 0$.

(a) When does the particle reach a velocity of 0 m/s? What is the significance of these value(s) of t?

(b) When does the particle have acceleration 0 m/s^2?

a) $v(t) = 6t - 3t^2 = 3t(2-t) = 0$ [redacted]

$\hookrightarrow t = 0, t = 2$ [redacted]

particle is stationary at values [redacted] $t = 0, 2$

b) $a(t) = 6 - 6t = 6(1-t) = 0$ [redacted]

$\hookrightarrow t = 1$ [redacted]

9.(10pts) Find the derivative of the following function using the definition of the derivative. **You will not receive any credit if you use any other method!**

$$f(x) = \frac{1}{2x-1}.$$

$$\lim_{h \to 0} \frac{\frac{1}{2(x+h)-1} - \frac{1}{2x-1}}{h}$$ [redacted]

$$= \lim_{h \to 0} \frac{2x-1 - 2(x+h) + 1}{h\,[2(x+h)-1]\,(2x-1)}$$ [redacted]

$$= \lim_{h \to 0} \frac{-2\cancel{h}}{\cancel{h}\,[(2(x+h)-1)(2x-1)]} = \frac{-2}{(2x-1)(2x-1)}$$

[redacted]

$$= \frac{-2}{(2x-1)^2}$$ [redacted]