1. (15 pts) Solve the following inequalities. Display your answer in interval notation.

a) \(|2 + 3x| \leq 3\)

\[-3 \leq 2 + 3x \leq 3\]

\[-5 \leq 3x \leq 1\]

\[-\frac{5}{3} \leq x \leq \frac{1}{3}\]

\([\frac{-5}{3}, \frac{1}{3}]\)

b) \(x^2 + 3x - 10 \geq 8\)

\(x^2 + 3x - 18 \geq 0\)

\((x+6)(x-3) \geq 0\)

\([\frac{1}{3}, \frac{1}{3}]\)

\([x \leq -6 \text{ or } x \geq 3]\)

\((-\infty, -6] \cup [3, \infty)\)
2. (15 pts) Find the limit if it exists. If the limit doesn’t exist explain why. Using L’Hopital’s Rule will not receive credit.

a) \[ \lim_{x \to 2} \frac{x^2 - 4}{x^2 + x - 6} = \lim_{x \to 2} \frac{(x-2)(x+2)}{(x-2)(x+3)} = \lim_{x \to 2} \frac{x+2}{x+3} = \frac{4}{5} \]

b) \[ \lim_{x \to 1} \frac{2-x}{(x-1)^2} = 0 \]

c) \[ \lim_{x \to 0} \frac{\sin(5x)}{\sin(8x)} = \lim_{x \to 0} \frac{5x \sin(5x)}{8x \sin(8x)} = \frac{5}{8} \lim_{x \to 0} \frac{\sin(5x)}{\sin(8x)} \\
\approx \frac{5}{8} \lim_{x \to 0} \frac{5x}{8x} = \frac{5}{8} \]

3. (10 pts) Use the Intermediate Value Theorem to show there is a root for the equation \( x^3 - x^2 + 3x - 5 = 0 \) between 1 and 2.

\[ P(x) = x^3 - x^2 + 3x - 5 \] is continuous for all reals.

\[ P(1) = 1 - 1 + 3 - 5 = -2 < 0 \]
\[ P(2) = 2^3 - 2^2 + 3(2) - 5 = 25 > 0 \]

Hence, by Intermediate Value Thm there exists a \( c \) between 1 and 2 such that \( P(c) = 0 \).
4. (10 pts) Find the equation of the tangent line to the curve \( y = x^2 - 2x + 3 \) at \( x = -1 \).

\[
y' = 2x - 2
\]

\[
y'(-1) = 2(-1) - 2 = -4.
\]

This is the slope of the tangent line to the curve at \( x = -1 \).

\[
y(-1) = (-1)^2 - 2(-1) + 3 = 6
\]

So, \((-1, 6)\) is a point on the curve.

Tangent line: \( y_T = mx + b \)

Where \( m = y'(-1) = -4 \).

\[
y_T = -4x + b
\]

Plug in pt. \((-1, 6)\):

\[
6 = -4(-1) + b \Rightarrow b = 2
\]

\[
y_T = -4x + 2
\]

5. (10 pts) Find the derivative of \( f(x) = \frac{1}{x+2} \) using the definition of derivative.

\[
f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}
\]

\[
= \lim_{h \to 0} \frac{\frac{1}{x+h+2} - \frac{1}{x+2}}{h}
\]

\[
= \lim_{h \to 0} \frac{\frac{1}{x+h+2} - \frac{1}{x+2}}{h} \cdot \frac{(x+h+2)(x+2)}{(x+h+2)(x+2)}
\]

\[
= \lim_{h \to 0} \frac{(x+2) - (x+h+2)}{h(x+h+2)(x+2)}
\]

\[
= \lim_{h \to 0} \frac{-h}{h(x+h+2)(x+2)}
\]

\[
= \lim_{h \to 0} \frac{-1}{(x+h+2)(x+2)} = \frac{-1}{(x+2)^2}
\]
6. (10 pts) Find each x-value at which \( f \) is discontinuous. Explain your answer fully using left and right-hand limits.

\[
f(x) = \begin{cases} 
  x+2 & \text{if } x < 0 \\
  2x^2 & \text{if } 0 \leq x \leq 1 \\
  3-x & \text{if } x > 1 
\end{cases}
\]

\[
\lim_{x \to 0^-} f(x) = 0 + 2 = 2 \\
\lim_{x \to 0^+} f(x) = 2(0)^2 = 0 \\
\lim_{x \to 1^-} f(x) = 2(1)^2 = 2 \\
\lim_{x \to 1^+} f(x) = 3 - 1 = 2 \\
\]

\( f(x) \) is discontinuous at \( x = 0 \) since
\[
\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x).
\]

7. (15 pts) Find \( \frac{dy}{dx} \) for the following:

a) \( y = ax^2 + \frac{b}{\sqrt{x^3}} + c \)

\[
y = ax^2 + b x^{-\frac{3}{2}} + c \\
\frac{dy}{dx} = 2ax - \frac{3b}{2} x^{-\frac{5}{2}}
\]

b) \( y = \sin \sqrt{x^2 + 1} \)

\[
y = \sin \left( x^2 + 1 \right)^{1/2} \\
\frac{dy}{dx} = \cos \left( x^2 + 1 \right)^{1/2} \cdot \frac{d}{dx} \left( x^2 + 1 \right)^{1/2} \\
= \cos \left( x^2 + 1 \right)^{1/2} \cdot \frac{1}{2} \left( x^2 + 1 \right)^{-1/2} \cdot \frac{d}{dx} \left( x^2 + 1 \right)
\]

\[
= \frac{2x \cos \sqrt{x^2 + 1}}{2 - \sqrt{x^2 + 1}}
\]

\[
= \frac{x \cos \sqrt{x^2 + 1}}{\sqrt{x^2 + 1}}
\]

\[
\frac{dy}{dx} = -x \sin \tan (5x) + 5 \sec^2 (5x) \cos x
\]

\[
= -x \sin x \tan (5x) + 5 \sec^2 (5x) \cos x
\]
8. (15 pts) Let \( f(x) = \frac{x^2 - 1}{|x - 1|} \quad (x \neq 1) \)

Write the above function \( f(x) \) as a piecewise function and find the following if they exist:

a) \( \lim_{x \to 1^-} f(x) \), \( \lim_{x \to 1^+} f(x) \), and \( \lim_{x \to 1} f(x) \).

b) Sketch \( f(x) \).

c) Where is \( f(x) \) continuous? Where is it differentiable?

d) Sketch \( f'(x) \).

\[
f(x) = \begin{cases} 
\frac{(x-1)(x+1)}{x-1} & \text{if } x > 1 \\
\frac{(x-1)(x+1)}{-(x-1)} & \text{if } x < 1 
\end{cases}
\]

\[
f(x) = \begin{cases} 
x+1 & \text{if } x > 1 \\
-x+1 & \text{if } x < 1 
\end{cases}
\]

\[
f(x) = \begin{cases} 
2 & \text{if } x > 1 \\
2 & \text{if } x < 1 
\end{cases}
\]

\[
f(x) = \begin{cases} 
1 & \text{if } x > 1 \\
-1 & \text{if } x < 1 
\end{cases}
\]