Zassenhaus Groups and Friends Conference 2025

# **Zassenhaus Groups and Friends Conference 2025**

May 31 - June 1, 2025

## Conference Schedule

A detailed program (in PDF format) is available here

Clicking on a name will take you to the talk's title below, and clicking on the title will display its abstract.

Time	Saturday 5/31	Sunday 6/1
8:00	Dogistration	
8:30	Registration	Riedl
9:00	Guzman	McCulloch
9:30	Donoven	Çınarcı
10:00	Tran	Coffee Break
10:30	Coffee Break	Zarrin
11:00	Lewis	Summers
11:30	Kirtland	Kappe
12:00	Klepadlo	Closing Remarks
12:20	Conference photograph	1
12:30		
1:00	Lunch Break	
1:30		
2:00	Foguel	
2:30	Russell	
3:00	Beike	
3:30	Coffee Break	
4:00	Feldman	
4:30	Martin	
5:00	Zaremsky	

# **Abstracts**

#### The isomorphism theorems for pointed g-digroups

Fernando Guzman (Binghamton University) Saturday 9:00 am

Digroups, and generalized digroups, g-digroups for short, have been considered as a generalization of continuous groups whose tangent space is a Leibniz algebra. This structure has been seen as a generalization of groups, therefore, efforts have been done to study properties and results that come from group theory, to explore if they hold in this new setting. A pointed g-digroup is a g-digroup with a distinguished bar-unit.

In this talk, we'll discuss the isomorphism theorems for pointed g-digroups, and show that the results for groups do extend to pointed g-digroups. Most of them also hold for g-digroups. This is joint work with Olga Patricia Salazar-Diaz.

#### Generation of simple vigorous groups of homeomorphisms

Casey Donoven (Montana State University Northern) Saturday 9:30 am

A group of homeomorphisms G of Cantor space is vigorous if for any clopen subsets  $B,C\subset A$  of Cantor space, there exists a  $\gamma\in G$  such that  $B\gamma\subseteq C$ . Bleak, Hyde, and Elliot (2024) proved that every finitely generated simple vigorous group is 2-generated. In this talk, I will present recent findings extending these results. For example, if G is a f.g. simple vigorous group, then (i) G is generated by 3 involutions; (ii) G is generated by an element of order G and G and G and G and G and G and G are initial generating set of size G for all G and G and G are initial element of G is contained in a generating pair. This is joint work with Collin Bleak, Scott Harper, and James Hyde.

## Algebraic Combinatorics meets Probability Theory: Vines and MAT-labeled graphs

Tan Tran (Binghamton University) Saturday 10:00 am

This talk explores the connection between two concepts from distinct areas of mathematics. The first concept, a vine, is a graphical model used to represent dependent random variables. Initially introduced by Joe (1994) and later formalized by Cooke (1997), vines have become an active research area with applications in probability theory and uncertainty analysis. The second concept, MAT-freeness, is a combinatorial property in the theory of freeness of the logarithmic derivation module of hyperplane arrangements. First studied by Abe-Barakat-Cuntz-Hoge-Terao (2016) and further developed by Cuntz-Muecksch (2020), MAT-freeness has been a topic of increasing interest. In particular, for graphic arrangements, Tsujie and I recently demonstrated that MAT-freeness is completely characterized by the existence of certain edge-labeled graphs, known as MAT-labeled graphs. I will show that there is a fascinating equivalence between the categories of locally regular vines and MAT-labeled graphs. Notably, this leads to an equivalence between the categories of regular vines and MAT-labeled complete graphs. This work is joint with H.M. Tran (Hanoi) and S. Tsujie (Hokkaido).

#### **Self-normalizing subgroups**

Mark Lewis (Kent State University) Saturday 11:00 am

We consider groups with few conjugacy classes of self--normalizing subgroups.

#### 2-covering numbers of finite groups

Joe Kirtland (Marist University) Saturday 11:30 am

A set of proper subgroups is a covering for a group G if its union is the whole group. The minimal number of subgroups needed to cover G is called the covering number of G and is denoted by  $\sigma(G)$ . A study of coverings of the Paige loop motivates the concept of a 2-covering for a group G, which is a set of proper subgroups of G such that every pair of elements of G are contained in at least one subgroup in the set. The minimal number of subgroups needed to 2-cover a group G is called the 2-covering number and denoted by  $\sigma_2(G)$ . Properties of 2-covering numbers will be presented with the 2-covering number determined for finite nilpotent groups, finite almost simple groups, and particular classes of finite solvable groups.

## Coverings of dihedral and permutation groups using centralizers

Matthew Klepadlo (Adelphi University) Saturday 12:00

A group is said to be covered if there exists proper subgroups such that their union is the same as the whole group. This paper will go into how we use centralizer subgroups to come up with coverings of smaller dihedral and permutation groups and obtain the "covering number" and "centralizer-covering number." We will also be

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highlighting a few notable theorems regarding coverings and use them to our advantage to finding said "covering/centralizer-covering number." The history of coverings and the interest mathematicians have in them will also be explored.

## Finite groups in which every subgroup of order divisible by p is normal

Tuval Foguel (Adelphi University) 2:00 pm

In this talk, I'll introduce two generalizations of Dedekind groups, called *PN*-groups and *PNQ*-groups. In *PN*-groups, every subgroup whose order is divisible by a fixed prime *p* is normal, while in *PNQ*-groups, such subgroups are permutable. We'll begin by showing that these groups must be either *p*'-groups or supersolvable. From there, I'll walk through a classification of both *PN*- and *PNQ*-groups. I'll end with a brief discussion of minimal non-*PN*-groups and some questions that remain open.

#### **Extended Springer fibers overview**

Amber Russell (Butler University) Saturday 2:30 pm

The Springer Correspondence associates to each irreducible representation of the Weyl group for a reductive Lie algebra a nilpotent orbit for the Lie algebra and an irreducible representation of the fundamental group of the Lie algebra. This result was due to T.A. Springer in the 1970s and is still providing fertile grounds of innovation today. The key tools in this work were the Springer resolution, a resolution of singularities for the nilpotent cone of the Lie algebra, and also careful study of the resulting Springer fibers. In the 1980s, George Lusztig expanded this to a bijection where all possible pairs of nilpotent orbits and irreducible representations of the fundamental group appear and the Weyl group is replaced by a class of new relative Weyl groups. This is Lusztig's Generalized Springer Correspondence.

Over the past several years, I have collaborated will William Graham and Martha Precup on a related project, spanning multiple publications with a new one currently being prepared. In particular, we have studied Extended Springer Fibers and connected them to Lusztig's Generalized Springer Correspondence in all classical types and relevant exceptional types. The goal of this presentation will be an overview of these results.

## p-Groups with derived length three and three character degrees

Nic Beike (Kent State University) Saturday 3:00 pm

We will construct examples of p-groups with derived length 3 and 3 character degrees. We will focus on groups of order  $p^6$ .

#### **Another dual to Schunck classes**

Arnold Feldman (Franklin and Marshall College) Saturday 4:00 pm

The duality of Fitting classes and formations has been widely studied, and Fitting classes and Schunck classes can also be considered to be dual in some sense. However, the definitions of Fitting and Schunck classes are not literally dual the way that those of Fitting classes and formations are. Here we identify a dual to Schunck classes of finite groups, which we call SchunckD classes, based on the standard definition of a Schunck class. We investigate properties and examples of Schunck classes and see how they differ from Fitting classes. The arguments are relatively elementary, and the topic could lend itself to exploration by advanced undergraduates.

#### Groups with a fixed character degree

Brandon Martin (Kent State University) Saturday 4:30 pm

Let  $x=d_1...d_m p_1^a...p_n^a$ , where the  $d_j$ 's and  $p_i$ 's are distinct primes, and  $a_i \in \mathbb{N}$  for all i. Let  $d=d_1...d_m$ . We've previously shown there exists a solvable group G, of order x, with  $d \in \operatorname{cd}(G)$  if and only if there is a sequence of congruences between the  $p_i$ 's and  $d_j$ 's where the product of the moduli of these congruences is precisely d. Now, we let relax the square-free condition on d and consider when an analogous result holds when  $x=d^mp^n$ .

#### Some difficult simple groups

Matt Zaremsky (University at Albany) Saturday 5:00 pm

Finite simple groups are famously classified, but infinite simple groups remain extremely mysterious in general. In particular, a famous conjecture of Boone and Higman predicts that every finitely generated group with solvable word problem embeds in a finitely presented simple group, so finitely presented simple groups are conjecturally ubiquitous, but actual examples are hard to come by. In this talk I will survey some of the bizarre and interesting (infinite) simple groups that arise, and mention some recent results, with a focus on a family of simple groups called twisted Brin-Thompson groups.

### Images of iterated commutators under group automorphisms

Jeffrey Riedl (University of Akron) Sunday 8:30 am

Let x, y be elements of a group G. For each integer  $m \ge 0$  let  $d_m$  denote the mth iterated commutator of x by y. Thus  $d_0 = x$ ,  $d_1 = [x,y]$ ,  $d_2 = [[x,y],y]$ , and so on. Let  $\mathbf{D} = \{d_0,d_1,d_2,...\}$  and suppose all the elements of  $\mathbf{D}$  commute with each other. Let  $\sigma$  be an automorphism of G such that  $x^\sigma = x$  and  $y^\sigma = y^u$  for some positive integer u. We establish a formula that expresses the image under  $\sigma$  of an arbitrary element of  $\mathbf{D}$  as a product of elements of  $\mathbf{D}$ .

We mention the application that motivated the establishment of this formula. Let p be a prime and let  $n \ge 2$ . The group  $U(p^n)$  of units modulo  $p^n$  acts naturally via automorphisms on the regular wreath product group  $W = Z_{p^2}$  wr  $Z_{pn}$ , and hence acts on the set  $\mathbf{N}$  consisting of all the normal subgroups of W that are contained in the base group of W. The formula enables the straightforward computation of the image  $\mathbf{N}^{\sigma}$  of an arbitrary  $N \in \mathbf{N}$  for an arbitrary automorphism  $\sigma \in U(p^n)$ .

#### **Groups with dense Chermak-Delgado subgroups**

Ryan McCulloch (Binghamton University) Sunday 9:00 am

Let X be a property pertaining to subgroups of a group. We say that a group G has dense X-subgroups if for each pair (H, K) of subgroups of G such that H < K and H is not maximal in K, there exists an X-subgroup X of G such that H < X < K. In this talk we consider groups with Chermak--Delgado dense subgroups and, more generally, with centralizer dense subgroups. This includes joint work with Marius Tarnauceanu.

#### Some results on derived length and character degrees

Burcu Çı narcı (Texas State University) Sunday 9:30 am

The character degrees of a finite group provide some important information about the structure of the group. A famous problem on the character degrees of a finite solvable group G is known as the Taketa problem and Isaacs-Seitz conjecture. This problem states that the inequality  $dI(G) \leq |cd(G)|$  holds for a finite solvable group G, where

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dI(G) is the derived length of G and |cd(G)| is the cardinality of the set of all irreducible character degrees of G. Although this conjecture is still open, many research articles have been published on this inequality. In this talk, we show that the Taketa inequality holds for G under some sufficient conditions.

## On the noncommuting set in infinite groups

Mohammad Zarrin (Texas State University) Sunday 10:30 am

Let G be a non-abelian group. A subset T of a group G is a set of pairwise noncommuting elements if xy&neq; yx for any two distinct elements x and y in T.

If  $|T| \ge |R|$  for any other set of pairwise noncommuting elements R in G, then T is called a maximal subset of pairwise noncommuting elements and the cardinality of such a subset (if it exists) is denoted by w(G). In this talk, among other things, we show that, for each positive integer m, there are only finitely many groups G, up to isoclinism, with w(G) = m, and we obtain similar results for groups with exactly m centralizers.

Also, we try to find the influence of the function w(G) on the structure of groups.

## On the number of disconnected character degree graphs satisfying Pálfy's inequality

Andrew Summers (Kent State University) Sunday 11:00 am

Let G be a finite solvable group with disconnected character degree graph  $\Delta(G)$ . Under these conditions, it follows from a result of Pálfy that  $\Delta(G)$  consists of two connected components. Another result of Pálfy's gives an inequality relating the sizes of these two connected components. In this talk, some background on character degree graphs and Pálfy's results will be presented. The number of possible component size pairs that satisfy Pálfy's inequality will be calculated. Additionally, for a fixed positive integer n, the number of distinct graph orders for which exactly n component size pairs satisfy Pálfy's inequality is shown.

## Element centralizers in a group centralizer lattice and centralizer-like subgroups

Luise-Charlotte Kappe (Binghamton University) Sunday 11:30 am

We note some properties of the centralizer map and recall the centralizer lattice of a group. Since the element centralizers generate all the other centralizers, we consider how the element centralizers sit in the lattice. We generalize this by considering the so-called centralizer-like subgroups of a group associated with a 2-letter word w(u,v). These are four subgroups defined by an operator that takes as input a subgroup H and returns the subgroup of group elements X such that W(X, h) = W(X

This is joint work with Wil Cocke,	Mark Lewis, and Ryan McCulloch.

Last updated: