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The Space of Geometric Realizations of Matroids

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I begin with an example-driven introduction to matroids, which generalize both matrices and graphs. They give an appropriate combinatorial structure to study finite collections of lines living in the plane. These line arrangements, as they are called, can also be studied using topology and algebraic geometry.

Zariski gave a pair of degree-six polynomials with the same types of singularities but whose complements have are non-homeomorphic. This motivates the search for similar “Zariski pairs” of line arrangements: two collections of lines with the same combinatorial intersection data but whose (complex projective) complements are not homeomorphic. Rybnikov produced one with thirteen lines in 1998 by gluing two smaller arrangements together. I will describe results from the literature that show that no such pair exists on nine lines or fewer. Amram, Sun, Teicher, Ye, Zarkh, and I investigate arrangements of ten lines. Students Liu and Buhmann, May, Shiyu, and I begin to investigate arrangements of eleven lines.
