Garry Bowlin (Oneonta)

Four Colors and Associativity

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The Four-Color Theorem was first proved by Appel and Haken in 1977 with the aid of a computer. Later, a second proof was given by Robertson, Sanders, Seymour, and Thomas. While the proof was simplified, it still relies on a computer in a significant way.

In 1990, Kauffman proved that the Four-Color Theorem is equivalent to the ability to find a non-trivial assignment of the 3-dimensional unit vectors **i**, **j**, and **k** to the variables of two associations of the multiple cross product $\mathbf{v}_1 \times \mathbf{v}_2 \times \cdots \times \mathbf{v}_n$, such that both associations have the same evaluation. (An assignment is *trivial* if it evaluates to zero.) The associations are determined by the map being colored.

Since elements of Thompson's group *F* represent instances of the associative law, one can prove that the Four-Color Theorem is equivalent to every element of *F*'s having a non-trivial assignment of the vectors **i**, **j**, and **k** for which that element's instance of associativity holds. I will prove that every positive element of *F* has such an assignment.

We call such elements *colorable*. I will consider several operations that preserve colorability of elements of *F*.
