Joanna Ellis-Monaghan (St. Michael's College)

The Tutte Polynomial and Potts Model in Statistical Mechanics

Abstract for the Colloquium 2008 November 13

What do beer foam, ghetto formation, and tumor migration have in common? They are all complex systems where microscale nearest-neighbor interactions (neighboring bubbles, neighboring residences, neighboring cells) determine macroscale properties. These macroscale phenomena can lead to bottling problems, segregation, or malignancy. How then do we model, and thereby predict, the behaviors of such systems?

Statistical mechanics provides one tool in the q-state Potts model. Its nascent form, the Ising model, allows two spins (up or down), and provides a model for magnetism. The Potts model with $q > 2$ allows q spins (thousands in the case of foam models). These models play important roles in the theory of phase transitions and critical phenomena in physics, and now have applications in every science.

The Potts model is typically constructed on lattices. When these lattices are viewed as graphs, then, remarkably, the Potts model is also equivalent to one of the most renowned graph invariants, the Tutte polynomial. Thus, there has been a remarkable synergy between the two fields in recent years. This talk will explore the Potts model, how it captures macroscale properties, its applications, and its relation to the Tutte and chromatic polynomials.
