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Solution of Blaschke's Problem on the Linearization of Planar Webs

Abstract for the Colloquium 2005 April 7

A d -web W in the plane consists of d one-parameter families of curves such that at any point A , the tangents to the d curves passing through A are in general position. A d -web is called *linear* if it is formed by d families of straight lines. It is called *linearizable* if it is equivalent to a linear d -web; i.e., if there exists a diffeomorphism of the plane, φ , such that $\varphi(W)$ is linear. Wilhelm Blaschke in the 1930s posed the problem of finding linearizability conditions for a d -web. He claimed that it is hopeless to find such conditions. Nevertheless, the problem was solved recently by the speaker in collaboration with M.A. Akivis and V.V. Lychagin.

For 3-webs, linearizability is equivalent to the existence of real, smooth solutions of a system of five algebraic equations of degrees not exceeding 16, 18, 18, 24, and 24. A 3-web is linearizable if and only if 1040 invariants of order not exceeding 9 (of which 18 are of order 8) vanish.

If a 4-web $W(4,2)$ is given by *web functions* $z = f(x,y)$ and $u = g(x,y)$, then its linearizability conditions are two PDEs of fourth order with respect to $f(x,y)$ and $g(x,y)$. Each of the PDEs contains 266 terms; they were obtained using the Mathematica package. Another form of the linearizability conditions is in terms of the curvature, K , and the basic invariant, a , of $W(4,2)$. The linearizability conditions are two equations expressing the covariant derivatives K_1 and K_2 of K in terms of K , a , and covariant derivatives of a up to the third order.

For d -webs with $d > 4$, the linearizability conditions are similar to those for 4-webs.

I will give numerous examples of applications of these conditions to known special classes of 4-webs. Also, I will present Mathematica codes for testing d -webs for linearizability and examples of their use.

PostScript abstract with references.

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