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Solution of Blaschke's Problem on the Linearization of Planar Webs

Abstract for the Colloquium 2005 April 7

A *d-web* W in the plane consists of d one-parameter families of curves such that at any point A, the tangents to the d curves passing through A are in general position. A d-web is called *linear* if it is formed by d families of straight lines. It is called *linearizable* if it is equivalent to a linear d-web; i.e., if there exists a diffeomorphism of the plane, φ, such that φ(W) is linear. Wilhelm Blaschke in the 1930s posed the problem of finding linearizability conditions for a d-web. He claimed that it is hopeless to find such conditions. Nevertheless, the problem was solved recently by the speaker in collaboration with M.A. Akivis and V.V. Lychagin.

For 3-webs, linearizability is equivalent to the existence of real, smooth solutions of a system of five algebraic equations of degrees not exceeding 16, 18, 18, 24, and 24. A 3-web is linearizable if and only if 1040 invariants of order not exceeding 9 (of which 18 are of order 8) vanish.

If a 4-web W(4,2) is given by *web functions* z = f(x,y) and u = g(x,y), then its linearizability conditions are two PDEs of fourth order with respect to f(x,y) and g(x,y). Each of the PDEs contains 266 terms; they were obtained using the Mathematica package. Another form of the linearizability conditions is in terms of the curvature, K, and the basic invariant, a, of W(4,2). The linearizability conditions are two equations expressing the covariant derivatives K_1 and K_2 of K in terms of K, a, and covariant derivatives of a up to the third order.

For d-webs with d > 4, the linearizability conditions are similar to those for 4-webs.

I will give numerous examples of applications of these conditions to known special classes of 4-webs. Also, I will present Mathematica codes for testing d-webs for linearizability and examples of their use.

PostScript abstract with references.
