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Homotopy in Biased Graphs: Combinatorics vs. Topology

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A path in the 1-skeleton of a topological cell complex (with endpoints in the 0-skeleton) is a sequence P = v_0, e_1, v_1, e_2, …, v_k. An elementary homotopy of P consists of replacing a subpath P' by another path Q', with the same endpoints, so that P' union Q' is contractible. One can require that P' union Q' is a circle, i.e., homeomorphic to a 1-sphere. Call this *topological homotopy*.

If we replace the 1-skeleton by an arbitrary graph and the condition of contractibility by a list of allowed circles in the graph, we have *combinatorial homotopy*. This is the sort of homotopy involved in my recent characterization of associative multary quasigroups. Here the list of allowed circles has to satisfy a “linearity” condition; the combination of the graph and the linear class of allowed circles is called a “biased graph”. A particular lemma in the proof of the quasigroup theorem displays clearly the operation of combinatorial homotopy.

The questions are: what does a topologist know (or want to know) about combinatorial homotopy, and how similar and how different are topological and combinatorial homotopy?