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# Biased Graphs and the Associative Law

## Abstract for the Combinatorics and Number Theory Seminar 2001 April 24

A quasigroup is like a group but without the identity, inverses, or associativity; all that is left is the multiplication table, which is an arbitrary Latin square. This is worth something: one has unique solvability of equations xy=z. Also, an identity can always be found. The crucial missing property is the associative law.

A *biased graph* is a graph together with a distinguished class of circles (a.k.a. circuits, cycles, polygons) that satisfies a certain combinatorial property. Each quasigroup with $m$ elements gives rise to a kind of biased graph called an $m$-fold biased expansion of $K_3$, the complete graph of order 3. Conversely, every $m$-fold biased expansion of $K_3$ is obtained from a quasigroup. Trying to generalize this construction of quasigroups to $K_n$ fails to be interesting because $K_4$ implies the associative law, so that a biased expansion of $K_n$ must come from a group if $n > 3$.

We explore this fact and possible ways of getting around it. For instance, an $m$-fold biased expansion of $C_n$ , the circle of length $n$, encodes an $n$-dimensional Latin hypercube, which might be considered the multiplication table of an ($n$-1)-ary operation. Chords in the circle imply specific associative properties which get stronger and stronger until when all chords have been added (so the graph becomes $K_n$) there is complete associativity, making the operation a group operation. What happens in between is completely unknown.