Spring 2018

- **January 22**
  Speaker: N/A
  Title: Organizational Meeting
  Abstract: We will discuss schedule and speakers for this semester

- **January 29**
  Speaker: Adrian Vasiu (Binghamton)
  Title: Purity of Crystalline Strata
  Abstract: We report on joint work with Jinghao Li on the purity of crystalline strata in characteristic p, such as Artin–Schreier strata, p-ranks strata, break points strata, and Newton polygon strata. This refines and reobtains prior works due to Zink, de Jong–Oort, Vasiu, and Yang and provides two new proofs of an unpublished result of Deligne.

- **February 5**
  Speaker: Alexander Borisov (Binghamton)
  Title: A hunt for the plane Keller maps
  Abstract: Plane Keller maps are the counterexamples to the two-dimensional Jacobian Conjecture. My approach to finding them (or proving that they do not exist) has been to compactify the input and output affine planes and to resolve the map. Smooth compactifications of the affine plane can be described by the graphs of curves at infinity. Over the past several years I found many natural restrictions on these graphs, but no “silver bullet” that would prevent the existence of Keller maps, thus proving the two-dimensional Jacobian Conjecture. And, indeed, the corresponding combinatorial problem actually has a solution! I will present this solution, which is a pair of trees with a “map” between them, and outline the approaches to the next challenge: finding the beast (Keller map) that “lives” on these trees.

- **February 12**
  Speaker: Sergio Da Silva (Cornell)
  Title: Frobenius splittings and the desingularization of hypersurfaces in positive characteristic
  Abstract: In its simplest form, a resolution of singularities is a birational map from a smooth algebraic variety to a singular one. Stronger versions require extra conditions such as the map being an isomorphism over the smooth locus. Hironaka's famous result provides an answer in characteristic zero, with various algorithmic approaches being later introduced. Desingularization in positive characteristic however has remained a difficult problem, mostly because characteristic zero techniques fail in this setting.
  I will give an overview of this desingularization algorithm and introduce Frobenius split varieties. Working in the affine hypersurface case, I will show why curves and surfaces that define Frobenius splittings can be desingularized without alteration to the algorithm. No prior experience with the resolution of singularities or Frobenius splittings is required.

- **February 26**
  Speaker: Sayak Sengupta (Binghamton)
  Title: Dimension Theory Part I
  Abstract: I would like to start with the graded rings and graded modules over graded rings, their properties, briefly go over the definition and uses of additive functions over Z; discuss Hilbert-Serre Theorem on Poincare Series on a graded module and ultimately lay the groundwork for the Dimension Theorem with some propositions.

- **March 12**

Speaker: Yuto Yamamoto (Yale)
Title: Tropical K3 surfaces
Abstract: Let Δ be a smooth reflexive polytope in dimension 3 and W be a tropical polynomial whose Newton polytope is its polar dual. By contracting a tropical K3 hypersurface defined by W, we can construct a 2-sphere equipped with an integral affine structure with singularities. We write the complement of the singularity as $i \colon B_0 \hookrightarrow B$, and the local system of integral tangent vectors on B_0 as T. The cohomology $H^1(B, i_\ast T)$ corresponds to the deformations of tropical structures of B. We show that there exists a primitive embedding of the Picard groupd of the toric variety associated with the normal fan of Δ into $H^1(B, i_\ast T)$.

- **March 19**
Speaker: Daniel Le (Toronto)
Title: The weight part of Serre's conjecture
Abstract: In the 70's, Serre conjectured that all odd irreducible continuous mod p Galois representations arise from modular forms. A decade later, he conjectured a recipe for the weight and level of the modular forms in terms of the Galois representations–a recipe which would play a key role in the proof of Fermat's Last Theorem. In Serre's original context, these conjectures are now known. We survey recent conjectures and results about the weight part of Serre's conjecture for more general automorphic forms. The main ingredient is a description of local Galois deformation rings using local models. This is joint work with B.V. Le Hung, B. Levin, and S. Morra.

- **March 26**
Speaker: Sayak Sengupta (Binghamton)
Title: Dimension Theory Part II
Abstract: We are going to start where we left off recalling the Hilbert-Serre Theorem and one of its consequences. Then we are going to introduce the concept of ideal-filtration of a ring and prove 3 results which eventually are going to lead us to the main result of the talk/; The Dimension Theorem (Proof included).

- **April 9**
Speaker: Jacob Matherne (UMass Amherst)
Title: Derived geometric Satake equivalence, Springer correspondence, and small representations
Abstract: A recurring theme in geometric representation theory is the ability to describe representations in terms of the topology of certain spaces. Two major theorems in this area are the geometric Satake equivalence and the Springer correspondence, which state:

1. For G a semisimple algebraic group, we can realize Rep(G) using intersection cohomology of the affine Grassmannian for the Langlands dual group.
2. For W a Weyl group, we can realize Rep(W) using intersection cohomology of the nilpotent cone.

In the late 90s, M. Reeder computed the Weyl group action on the zero weight space of the irreducible representations of G, thereby relating Rep(G) to Rep(W). More recently, P. Achar, A. Henderson, and S. Riche established a functorial relationship between the two phenomena above. In my talk, I will review this story and discuss a result which extends their functorial relationship to the setting of mixed, derived categories.

- **April 16**
Speaker: Changwei Zhou (Binghamton)
Title: Gaussian measure and discrete Laplacian
Abstract: The Poisson summation formula for Gaussians played an important role for 1D Arakelov theory. The analog of Gaussian measure in 2D Arakelov theory has been absent. In this talk we shall discuss some attempts to

construct Gaussian measures in 2D setting using discrete Laplacian. Specifically we want to discuss a “no free lunch theorem” for discrete Laplacian. If time allows, we may talk about the connection between discrete Laplacian, discrete holomorphic derivative as well as the discrete Gaussian free field.

- **April 23**
  Speaker: Micah Loverro (Binghamton)
  Title: G-modules and Lie(G)-modules
  Abstract: I'll continue from my previous talk, where we have a finite dimensional representation V of a semisimple simply-connected algebraic group G_K, and we want to know when a Lie(G)-module M inside V is also a G-module, where G=Spec(A) is a smooth affine group scheme over a Noetherian domain R whose field of fractions is K, and G_K = Spec(A \otimes_R K). We have a condition on the highest weight of the representation which guarantees that such an M is a G-module. This time I'll show how to construct counterexamples when the condition is not met.

- **April 30**
  Speaker: Brian Hwang (Cornell)
  Title: Local Models of Shimura Varieties and Limit Linear Series on Algebraic Curves
  Abstract: Roughly speaking, Shimura varieties are moduli spaces of abelian varieties (with some additional structure) that are closely linked to the structure of reductive groups. They have demonstrated themselves as a useful tool for studying number theoretic questions as well as possessing an interesting geometry in and of themselves. However, their mod p reductions are notoriously difficult to study. One fruitful way to study these mod p reductions is by using local models, which can be patched together to obtain the entire space. Recently, we discovered that local models of certain Shimura varieties also arise in a different subject: limit linear series on algebraic curves, which concerns degenerations of line bundles on algebraic curves. This simplifies certain aspects of local models and highlights some strange connections between the two subjects that we will explain. This is joint work with Binglin Li.

- **May 7**
  Speaker: Patrick Milano (Binghamton) – Thesis Defense
  Title: Mixed Ghost Spaces
  Abstract: An Arakelov divisor D on a number field K is a formal finite sum of prime ideals of O_K and infinite primes of K. A ghost space is a kind of object introduced by Borisov in order to define $H^0(D)$ and $H^1(D)$ for Arakelov divisors.
  A mixed ghost space is a generalization of the ghost spaces used to construct $H^0(D)$ and $H^1(D)$. I will develop the theory of mixed ghost spaces. The main focus will be on ghost spaces given by Gaussian functions on $\mathbb{R}^n$, which can be completely classified up to some natural equivalence relations. I will also use mixed ghost spaces to construct long exact sequences in Borisov's cohomology.