The Algebra Seminar

In a normal semester, the seminar would meet on Tuesdays in room WH-100E at 2:50 p.m. There would be refreshments served at 4:00 in room WH-102. But under the current COVID-19 pandemic, in-person seminar talks do not seem safe, and no refreshments or socializing in person will be allowed in Fall 2020.

We therefore propose that all speakers at the Algebra Seminar present their talks using Zoom. Anyone wishing to give a talk in the Algebra Seminar this semester is requested to contact the organizers at least one week ahead of time, to provide a title, abstract and a Zoom meeting link for posting on this webpage.

Organizers: Alex Feingold and Hung Tong-Viet

To receive announcements of seminar talks by email, please join the seminar's mailing list.

Zoom link: https://binghamton.zoom.us/j/95069915454

Please note that we have now implemented a Passcode required to join the zoom meeting. That passcode is the number obtained by adding up all the positive integers from 1 to 100, 1+2+…+100. That should keep the zoom bombers out!

Fall 2020

- **September 1**
  Organizational Meeting

  Join Zoom Meeting using this link. The Meeting ID: 821 340 7154. Please think about giving a talk in the Algebra Seminar, or inviting an outside speaker.

- **September 8**
  Fikreab Solomon Admasu (Binghamton University)
  Composition laws from Gauss to Bhargava

  Abstract: One of the fundamental objects of interest in number theory is the composition law of binary quadratic forms. In this talk, we will start with a review of the original complicated formulation due to Gauss and then discuss subsequent simplifications by Dirichlet, et al. Another approach involves identifying equivalence classes of binary quadratic forms with ideal classes in a quadratic ring and using the natural group structure of the ideal class group to formulate the composition law of binary quadratic forms. After about 200 years, M. Bhargava gave a new perspective on Gauss' composition law using so-called 2x2x2 cubes of integers and derived more composition laws. We will discuss the main theorems in 'Higher Composition Laws I' by Bhargava and mention the applications if time permits.

- **September 15**
  Fikreab Solomon Admasu (Binghamton University)
  Composition laws from Gauss to Bhargava

  Abstract: Continuation of the talk from September 8, with focus on the newer Bhargava approach.

- **September 22**
  Nguyen Ngoc Hung (University of Akron)
  Fields of values of group characters

  Abstract : Representation theory and character theory have been developed as tools to study structure of finite groups. I will present some results, both old and new, on fields of character values. In particular, I will present an extension of results of Burnside and Navaro-Tiep on the existence of real/rational irreducible characters in even-order groups.

- **September 30, 7:30 PM**
  Burkard Polster (Monash University, Melbourne, Australia)
  Mathologer on YouTube

  Abstract: Burkard Polster (aka the Mathologer) will talk about his mathematical YouTube channel and discuss interesting questions submitted to him by the audience. This zoom talk will be live from Melbourne, Australia, so in order to accommodate the large time zone difference between Australia and New York, it will take place on Wednesday evening at 7:30 PM (Binghamton time zone).

- **October 6**
  Per Kurlberg (KTH Stockholm)
  Class numbers and class groups for definite binary quadratic forms

  Abstract: Gauss made the remarkable discovery that the set of integral binary quadratic forms of fixed discriminant carries a composition law, i.e., two forms can be “glued together” into a third form. Moreover, as two quadratic forms related to each other via an integral linear change of variables can be viewed as equivalent, it is natural to consider equivalence classes of quadratic forms. Amazingly, Gauss' composition law makes these equivalence classes into a finite abelian group - in a sense it is the first abstract group “found in nature”. Extensive calculations led Gauss and others to conjecture that the number h(d) of equivalence classes of such forms of negative discriminant d tends to infinity with |d|, and that the class number is h(d) = 1 in exactly 13 cases: d is in { -3, -4, -7, -8, -11, -12, -16, -19, -27, -28, -43, -67, -163 }. While this was known assuming the Generalized Rieman Hypothesis, it was only in the 1960's that the problem was solved by Alan Baker and by Harold Stark. We will outline the resolution of Gauss' class number one problem and survey some known results regarding the growth of h(d). We will also consider some recent conjectures regarding how often a fixed abelian group occur as a class group, and how often an integer occurs as a class number. In particular: do all abelian groups occur, or are there “missing” class groups?

- **October 13**
  Mark Lewis (Kent State University)
  Finite, solvable, tidy groups (or how I spent my summer vacation.)

  Abstract: Let G be a group and let x be an element of G. We define ${\rm Cyc}_G (x) = \{ g \in G \mid \langle g, x \rangle {\rm ~is~ cyclic} \}$. It is easy to see that ${\rm Cyc}_G (x)$ is not necessarily a subgroup of G. We say that a group G is tidy if ${\rm Cyc}_G (x)$ is a subgroup for all $x \in G$. We will find a classification of finite, tidy p-groups and finite, tidy $\{ p, q \}$-groups for primes p and q. We will see how these can be used to characterize finite, solvable, tidy groups. This includes work that was done during the 2020 Summer REU at Kent State University. Students involved were Nicholas F. Beike, Colin Heath, Kaiwen Lu, and Jamie D. Pearce. The graduate assistants were Rachel Carleton and David Costanzo.

- **October 20**
  No seminar

- **October 27**
  Hung Tong-Viet (Binghamton University)
  Proportions of vanishing elements in finite group

  Abstract: An element g of a finite group G is called a \emph{vanishing element} of G if there exists an irreducible complex character χ of G such that $\chi(g)=0$. In this case, the element g is also called a zero of character. Zeros of characters play an important role in block theory as well as in representation theory. In this talk, I will discuss the influence of vanishing elements on the structure of finite groups and will focus on bounding the proportion of vanishing elements. In particular, I will show that the proportion of vanishing elements of every finite non-abelian group is bounded below by $1/2$ and classify all finite groups whose proportions of vanishing elements attain this bound. Moreover, if this proportion is less than or equal to $2/3$, then the group must be solvable. (This is joint work with Lucia Morotti).

- **November 3**
  Samantha Wyler (Kent State University)
  Special, Extra Special, Semi Extra Special, and Ultra Special Groups

  Abstract: A non-abelian finite p-group is special if the commutator subgroup, the center, and the Frattini subgroup are equal. In this talk, we will discuss some properties of Special groups, Extra Special groups, and Semi Extra Special groups. Semi Extra Special Groups can be thought of as a generalization of Extra Special groups. Additionally, every Semi Extra Special group is a Special group, and Semi Extra Special groups are also a generalization of Ultra Special groups. This talk will largely be based on Professor Mark Lewis's paper "Semi-extra special Groups."

- **November 10**
  Zach Costanzo (Binghamton University)
  Groups whose real-valued character degrees are all prime powers

  Abstract: Let G be a finite group. Dolfi, Pacifici, and Sanus show that if all of the real-valued irreducible characters of G have prime degree, then G is solvable and the real-valued characters are contained in a subset of $\{1, 2, p\}$ for some odd prime p. Here, we attempt to generalize some of these results by assuming instead that all of the real-valued irreducible characters of G have prime power degrees. We classify such groups in the case that G is non-solvable, and show a similar result about set of real-valued character degrees in the case that G is solvable.

- **November 17**
  Jonathan Doane (Binghamton University)
  Affine clone of Boole

  Abstract: A Boolean operation is a function $f:\{0,1\}^n\to \{0,1\}$ ($n\in \mathbb{N}$), and it is called affine if it can be expressed as $f(x_1,\ldots, x_n) = c_0+c_1x_1+\cdots+c_nx_n$ ($\{c_i\}_{i=0}^n \subseteq \{0,1\}$) where addition is taken mod 2. In this talk, we will show how the set of all affine Boolean operations arises naturally (as the ``clone'' of an algebra) when explore a generalization of ring (with unity) theory and bounded lattice theory.

- **November 24**
  No seminar

- **December 1**
  Dikran Karagueuzian (Binghamton University)
  Moments of Polynomials over Finite Fields

  Abstract: A polynomial over a finite field can be regarded as a map of the finite field to itself. The variance of the inverse image sizes has been studied in connection with the question of whether such maps are a good substitute for random maps. We are able to show that polynomial maps are not random in the sense that all moments, not just the variance, must be integers, in an asymptotic sense as the size of the finite field becomes large. This is based on joint work with Per Kurlberg.

- **December 8**
  No seminar
