Summer Challenge (due August 31)

Fix a circle \$\Gamma\$. Let \$T=\triangle ABC\$ be a triangle inscribed in the circle \$\Gamma\$ which is not a right triangle and let \$H\$ be the orthocenter of \$T\$. The line \$\HA\$ intersects the circle \$\Gamma\$ at \$A\$ and at a second point \$A_1\$ (which can be \$A\$ if the line is tangent to the circle). Likewise, the line \$\HB\$ intersects \$\Gamma\$ at a second point \$B_1\$, and the line \$\HC\$ intersects \$\Gamma\$ at a second point \$C_1\$. The triangle \$\triangle A_1B_1C_1\$ is again inscribed in \$\Gamma\$. We denote this triangle by \$\Phi(T)\$. Warning: \$\Phi(T)\$ can be a right triangle.

- I a) Show that triangles \$T\$ and \$\Phi(T)\$ are congruent if and only if either \$T\$ is equilateral or the angles of \$T\$ are \$\pi/7\$, \$2\pi/7\$, \$4\pi/7\$.
- b) For every integer k>0 find the number t_k of non-congruent triangles T inscribed in $\$ are congruent. Here ρ^k denotes the composition ρ^k in ρ^k in
- $_{\rm I}$ c) Is it true that if $\Phi(T)$ and T are congruent then $\Phi(T)=T$ for some m?
- d) Find and prove your own results about \$\Phi\$.

From:

https://www2.math.binghamton.edu/ - **Department of Mathematics and Statistics, Binghamton University**

Permanent link:

https://www2.math.binghamton.edu/p/pow/summer_challenge

Last update: 2020/08/31 05:30

×