Problem 7 (due Monday, December 6)

A universe consists of an infinite collection of galaxies G_i indexed by the integers. Each galaxy G_i consists of a finite number g_i of stars. Every star in galaxy G_i is connected with every star in galaxy G_{i-1}, with every star in galaxy G_{i+1}, possibly with some stars in galaxy G_i, and with no other stars. It is known that for every sufficiently large n the number $g_{-n}+g_{-n+1}+\ldots +g_{n-1}+g_{n}$ does not exceed $n^{3/2}$. Each star has mass equal to the arithmetic mean (i.e. the average) of the masses of all the stars connected with it. Prove that all stars have the same mass.

We received a partial solution from Pluto Wang, who showed that all stars in a given galaxy have the same mass, and a complete solution from Ashton Keith. Ashton's solution follows the same ideas as our solution, though some details are different. For details see the following link Solution.