Problem 6 (due Monday, November 20)

Find all bounded continuous functions $f:\mathbb R\longrightarrow \mathbb R$ which satisfy the following condition: \[f^2(x)-f^2(y)=f(x+y)f(x-y)\ \ \text{for all $x,y\in \mathbb R$}.\] Here $f^2=f\cdot f$ is the square of the function f (and not the composition of f with itself).

We received two attempts at solving the problem, both far from being correct. The functions which satisfy the conditions of the problem are exactly all functions of the form $f(x)=A\sin bx$ for some real numbers A,b. For detailed solutions (we have two) see the following link Solution.