Problem 5 (due Monday, April 14)

A positive integer N has the following properties:

(1) N is a square.

(2) N is the sum of two positive squares in a unique way (up to order).

(3) $\text{d}(N)\phi(N)=8N$.

(d(N) is the number of positive divisors of N and ϕ is the Euler function).

What is N?

The answer is $N=2025$. I created the problem as a New Year puzzle to welcome the year 2025. We received solutions from David Biddle, Raisha Chowdhury, Robert Kroplewski, Emily (Qingyue) Liu, Josiah Moltz. One solver submitted incorrect answer, another solver submitted a correct answer without justification that it is the unique possible answer. The other 3 solutions were complete and they are essentially the same as our in-house solution. For a detailed solution see the following link Solution.