Problem 5 (due Monday, April 26)

The sequence (a_n) is defined recursively as follows: $a_1=1$, $a_{n+1}=\sin a_n$. Prove that the sequence $(\sqrt{n}a_n)$ converges and find its limit.

Two solutions were received, from John Mignone and Yuqiao Huang. Both are correct and are based on a result going back to Cauchy, which is a special case of Stolz theorem (solution by Yuqiao) and is related to Cesaro summation (John's solution). This is covered in our second solution. Our first solution is more direct (but longer) and shows that the sequence $\sqrt{n}a_n$ is increasing. For details see the following link Solution.